1. Advanced Architectures
2. Carrier Phase Modulation
3. Chapter 3:Electromechanical-Energy-Conversion
4. Chapter 4: Introduction to Rotating Machines

Advanced Architectures

1. Introduction

In the previous sections of this course. we have concentrated on singleprocessor architectures and techniques to improve upon their performance, such as:

– Efficient algebraic hardware implementations

– Enhanced processor operation through pipelined instruction execution and multiplicity of functional units

– Memory hierarchy

– Control unit design

– I/O operations

Through these techniques and implementation improvements, the processing power of a computer system has increased by an order of magnitude every 5 years. We are (still) approaching performance bounds due to physical limitations of the hardware.

- Several approaches of parallel computer are possible

– Improve the basic performance of a single processor machine

Architecture / organization improvements

Implementation improvements

SSI --> VLSI --> ULSI

Clock speed

Packaging

– Multiple processor system architectures

Tightly coupled system

Loosely coupled system

Distributed computing system

- Parallel computer: SIMD computer, MIMD computer

2. Multiple Processor Systems

System with multiprocessor CPUs can be divided into multiprocessor and multicomputers. In this section we will first study multiprocessors and then multicomputers

Shared-Memory Multiprocessor

A parallel computer in which all the CPUs share a common memory is called a tightly coupled systems

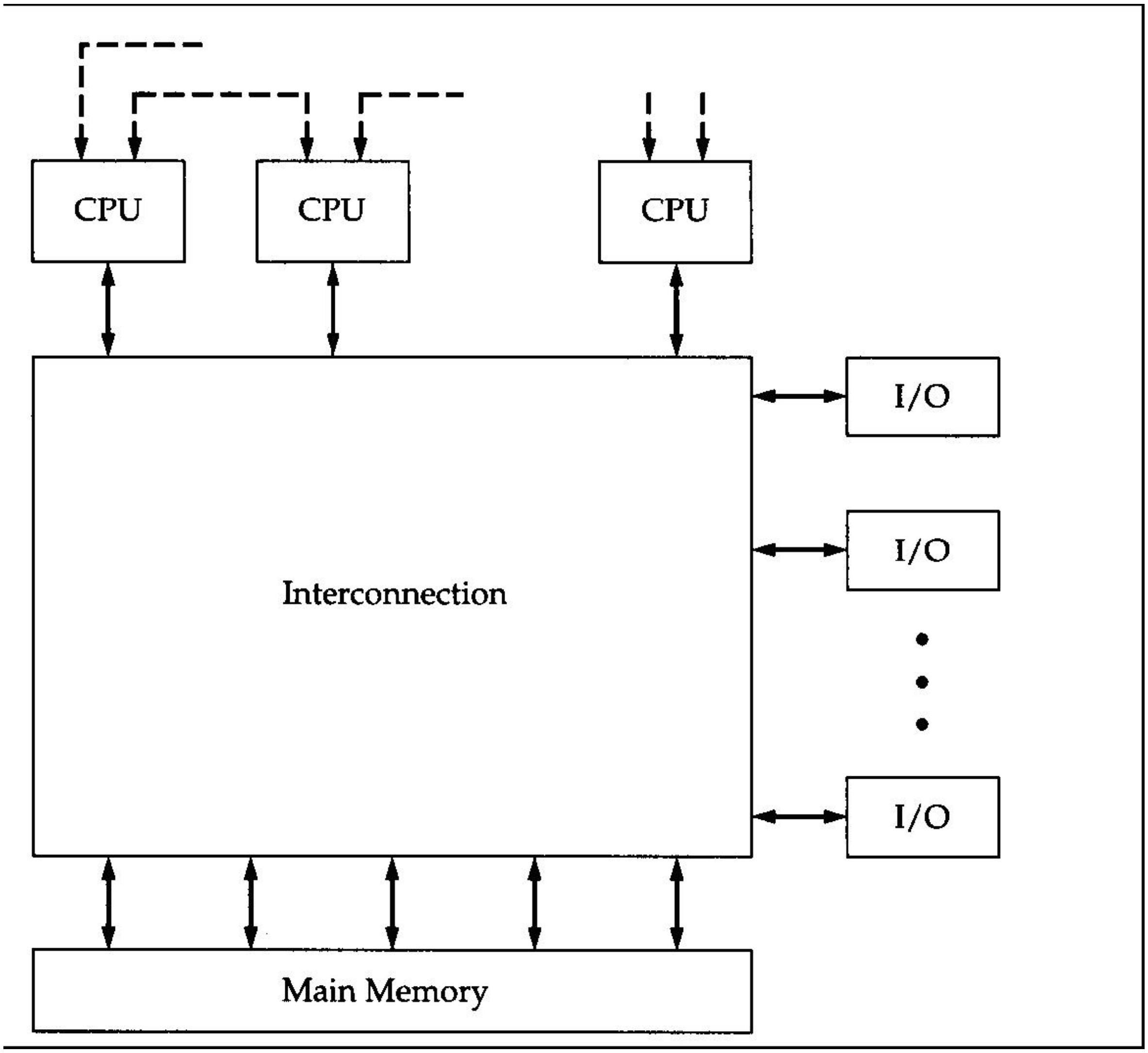


Figure 16.1. Tightly coupled systems, Shased-memory multiprocessor

- The features of the system are as follow.

– Multiple processors

– Shared, common memory system

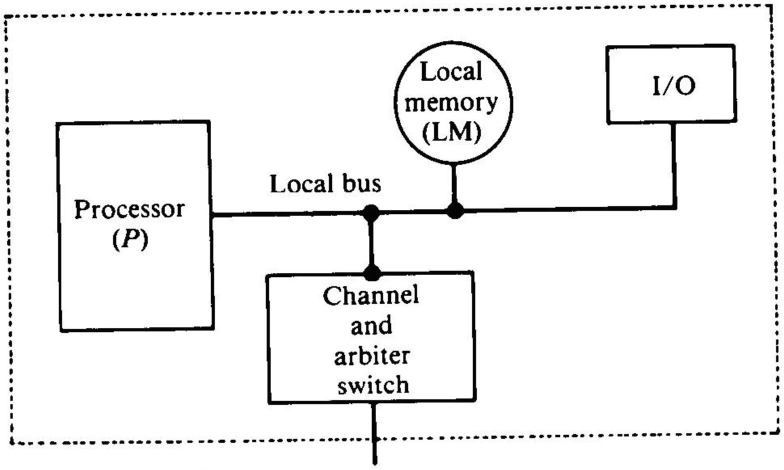
– Processors under the integrated control of a common operating system

– Data is exchanged between processors by accessing common shared variable locations in memory

– Common shared memory ultimates presents an overall system bottleneck that effectively limits the sizes of these systems to a fairly small number of processors (dozens)

Message-passing multiprocessor

A parallel computer in which all the CPUs has a local independent memory is called a loosely coupled systems



(a) A computer module

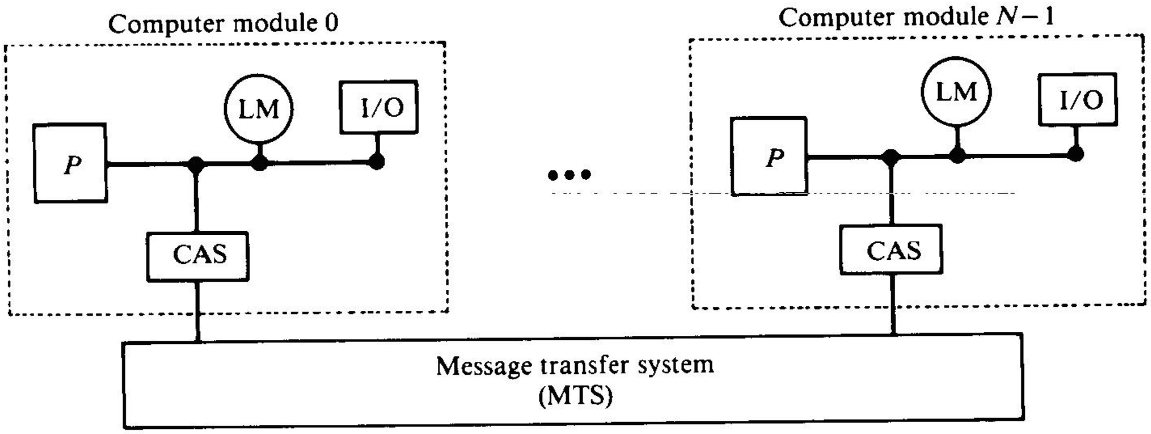


Figure 16.2. Loosely coupled systems, Message-passing multiprocessor

- The features of the system are as follow.

– Multiple processors

– Each processor has its own independent memory system

– Processors under the integrated control of a common operating system

– Data exchanged between processors via interprocessor messages

– This definition does not agree with the one given in the text

Distributed computing systems

Now we can see the message-passing computer that multicomputer are held togerther by network.

– Collections of relatively autonomous computers, each capable of independent operation

– Example systems are local area networks of computer workstations

+ Each machine is running its own "copy" of the operating system

+ Some tasks are done on different machines (e.g., mail handler is on one machine)

+ Supports multiple independent users

+ Load balancing between machines can cause a user's job on one machine to be shifted to another

Performance bounds of Multiple Processor Systems

- For a system with n processors, we would like a net processing speedup (meaning lower overall execution time) of nearly n times

when compared to the performance of a similar uniprocessor system
- A number of poor performance “upper bounds” have been proposed over the years

Maximum speedup of O(log n)

Maximum speedup of O(n / ln n)

- These “bounds” were based on runtime performance of applications and were not necessarily valid in all cases
- They reinforced the computer industry’s hesitancy to “get into” parallel processing

3. Parallel Processors

The machines are the true parallel processors (also called concurrent processors)

These paralle machines fall into Flynn’s taxonomy classes of SIMD and MIMD systems

– SIMD: Single Instruction stream and Multiple Data streams

– MIMD: Multiple Instruction streams and Multiple Data streams

SIMD Overview

- Single “control unit” computer and anarray of “computational” computers
- Control unit executes control-flowinstructions and scalar operations and passes vector instructions to the processor array
- Processor instruction types:

– Extensions of scalar instructions

Adds, stores, multiplies, etc. become vector operations executed in all processors concurrently

– Must add the ability to transfer vector and scalar data between processors to the instruction set -- attributes of a “parallel language”

- SIMD Examples

Vector addition

C(I) = A(I) + B(I)

Complexity O(n) in SISD systems for I=1 to n do

C(I) = A(I) + B(I)

Complexity O(1) in SIMD systems

Matrix multiply

A, B, and C are n-by-n matrices

Compute C= AxB

Complexity O(n3) in SISD systems

n2 dot products, each of which is O(n)

Complexity O(n2) in SIMD systems

Perform n dot products in parallel across M the array

Image smoothing

– Smooth an n-by-n pixel image to reduce “noise”

– Each pixel is replaced by the average of itself

and its 8 nearest neighbors

– Complexity O(n2) in SISD systems

– Complexity O(n) in SIMD systems

Pixel and 8 neighbors

MIMD Systems Overview

- MIMD systems differ from SIMD ones in that the “lock-step” operation requirement is removed
- Each processor has its own control unit and can execute an independent stream of

instructions

– Rather than forcing all processors to perform the same task at the same time, processors can be assigned different tasks that, when taken as a whole, complete the assigned application

- SIMD applications can be executed on an MIMD structure

– Each processor executes its own copy of the SIMD algorithm

- Application code can be decomposed into communicating processes

– Distributed simulations is a good example of a

very hard MIMD application

- Keys to high MIMD performance are

– Process synchronization

– Process scheduling

- Process synchronization targets keeping all processors busy and not suspended awaiting data from another processor

- Process scheduling can be performed

– By the programmer through the use of parallel language constructs

Specify apriori what processes will be instantiated and where they will be

executed

– During program execution by spawning processes off for execution on available processors.

Fork-join construct in some languages

- System examples

SIMD

– Illiac IV

One of the first massively parallel systems 64 processors

– Goodyear Staran: 256 bit-serial processors

– Current system from Cray Computer Corp.uses supercomputer (Cray 3) front end coupled to an SIMD array of 1000s of processors

MIMD

– Intel hypercube series:

Supported up to several hundred CISC processors

Next-gen Paragon

– Cray Research T3D

Cray Y-MP coupled to a massive array of Dec Alphas

Target: sustained teraflop performance

4. Discussions

- Problems

– Hardware is relatively easy to build

– Massively parallel systems just take massive amounts of money to build

– How should/can the large numbers of processors be interconnected

– The real trick is building software that will exploit the capabilities of the system

- Reality check:

– Outside of a limited number of high-profile applications, parallel processing is still a“young” discipline

– Parallel software is still fairly sparse

– Risky for companies to adopt parallel strategies, just wait for the next new SISD system.

Carrier Phase Modulation

Phase Shift Keying (PSK)

Information is impressed on the phase of the carrier. As data changes from symbol period to symbol period, the phase shifts.

Equation:

$$\forall m, m \in \{1, 2, \ldots, M\} : \left(s_m(t) = AP_T(t)\cos\left(2\pi f_c t + \frac{2\pi (m-1)}{M}\right)\right)$$

Example:

Binary $s_1(t)$ or $s_2(t)$

Representing the Signals

An orthonormal basis to represent the signals is

Equation:

$$\psi_1(t) = \frac{1}{\sqrt{E_s}} AP_T(t)\cos(2\pi f_c t)$$

Equation:

$$\psi_2(t) = \frac{-1}{\sqrt{E_s}} AP_T(t)\sin(2\pi f_c t)$$

The signal

Equation:

$$S_m(t) = AP_T(t)\cos\left(2\pi f_c t + \frac{2\pi (m-1)}{M}\right)$$

Equation:

$$S_m(t) = A\cos\left(\frac{2\pi (m-1)}{M}\right)P_T(t)\cos(2\pi f_c t) - A\sin\left(\frac{2\pi (m-1)}{M}\right)P_T(t)\sin(2\pi f_c t)$$

The signal energy

Equation:

$$\begin{aligned} E_s &= \int_{-\infty}^{\infty} A^2 {P_T}^2(t)\cos^2\left(2\pi f_c t + \tfrac{2\pi(m-1)}{M}\right) \mathrm{d}\, t \\ &= \int_0^T A^2 \left(\tfrac{1}{2} + \tfrac{1}{2}\cos\left(4\pi f_c t + \tfrac{4\pi(m-1)}{M}\right)\right) \mathrm{d}\, t \end{aligned}$$

Equation:

$$E_s = \frac{A^2 T}{2} + \frac{1}{2} A^2 \int_0^T \cos\left(4\pi f_c t + \frac{4\pi(m-1)}{M}\right) \mathrm{d}\, t \simeq \frac{A^2 T}{2}$$

(Note that in the above equation, the integral in the last step before the aproximation is very small.) Therefore,
Equation:

$$\psi_1(t) = \sqrt{\frac{2}{T}} P_T(t) \cos(2\pi f_c t)$$

Equation:

$$\psi_2(t) = \left(-\sqrt{\frac{2}{T}}\right) P_T(t) \sin(2\pi f_c t)$$

In general,
Equation:

$$\forall m, m \in \{1, 2, \ldots, M\} : \left(s_m(t) = A P_T(t) \cos\left(2\pi f_c t + \frac{2\pi(m-1)}{M}\right)\right)$$

and $\psi_1(t)$
Equation:

$$\psi_1(t) = \sqrt{\frac{2}{T}} P_T(t) \cos(2\pi f_c t)$$

Equation:

$$\psi_2(t) = \sqrt{\frac{2}{T}} P_T(t) \sin(2\pi f_c t)$$

Equation:

$$s_m = \begin{pmatrix} \sqrt{E_s} \cos\left(\frac{2\pi(m-1)}{M}\right) \\ \sqrt{E_s} \sin\left(\frac{2\pi(m-1)}{M}\right) \end{pmatrix}$$

Demodulation and Detection

Equation:

$$r_t = s_m(t) + N_t, \text{ for some} m \in \{1, 2, \ldots, M\}$$

We must note that due to phase offset of the oscillator at the transmitter, **phase jitter** or **phase changes** occur because of propagation delay.
Equation:

$$r_t = AP_T(t)\cos\left(2\pi f_c t + \frac{2\pi\,(m-1)}{M} + \varphi\right) + N_t$$

For binary PSK, the modulation is antipodal, and the optimum receiver in AWGN has average bit-error probability

Equation:

$$\begin{aligned} P_e &= Q\left(\sqrt{\tfrac{2(E_s)}{N_0}}\right) \\ &= Q\left(A\sqrt{\tfrac{T}{N_0}}\right) \end{aligned}$$

The receiver where

Equation:

$$r_t = \pm(AP_T(t)\cos(2\pi f_c t + \varphi)) + N_t$$

The statistics

Equation:

$$\begin{aligned} r_1 &= \int_0^T r_t \alpha\cos\left(2\pi f_c t + \hat{\varphi}\right)\,\mathrm{d}\,t \\ &= \pm\left(\int_0^T \alpha A\cos(2\pi f_c t + \varphi)\cos\left(2\pi f_c t + \hat{\varphi}\right)\,\mathrm{d}\,t\right) + \int_0^T \alpha\cos\left(2\pi f_c t + \hat{\varphi}\right)N_t\,\mathrm{d}\,t \end{aligned}$$

Equation:

$$r_1 = \pm\left(\frac{\alpha A}{2}\int_0^T \cos\left(4\pi f_c t + \varphi + \hat{\varphi}\right) + \cos\left(\varphi - \hat{\varphi}\right)\,\mathrm{d}\,t\right) + \eta_1$$

Equation:

$$r_1 = \pm\left(\frac{\alpha A}{2}T\cos\left(\varphi - \hat{\varphi}\right)\right) + \int_0^T \pm\left(\frac{\alpha A}{2}\cos\left(4\pi f_c t + \varphi + \hat{\varphi}\right)\right)\,\mathrm{d}\,t + \eta_1 \pm \left(\frac{\alpha AT}{2}\cos\left(\varphi - \hat{\varphi}\right)\right) + \eta_1$$

where $\eta_1 = \alpha\int_0^T N_t\cos\left(\omega_c t + \hat{\varphi}\right)\,\mathrm{d}\,t$ is zero mean Gaussian with variance $\simeq \frac{\alpha^2 N_0 T}{4}$.

Therefore,

Equation:

$$\begin{aligned} \overline{P_e} &= Q\left(\frac{2\frac{\alpha AT}{2}\cos\left(\varphi - \hat{\varphi}\right)}{2\sqrt{\frac{\alpha^2 N_0 T}{4}}}\right) \\ &= Q\left(\cos\left(\varphi - \hat{\varphi}\right)A\sqrt{\frac{T}{N_0}}\right) \end{aligned}$$

which is not a function of α and depends strongly on phase accuracy.

Equation:

$$P_e = Q\left(\cos\left(\varphi - \hat{\varphi}\right)\sqrt{\frac{2E_s}{N_0}}\right)$$

The above result implies that the amplitude of the local oscillator in the correlator structure does not play a role in the performance of the correlation receiver. However, the accuracy of the phase does indeed play a major role. This point can be seen in the following example:

Example:
Equation:

$$x_{t'} = -1^{i} A \cos\left(-\left(2\pi f_c t'\right) + 2\pi f_c \tau\right)$$

Equation:

$$x_t = -1^{i} A \cos\left(2\pi f_c t - \left(2\pi f_c \tau' - 2\pi f_c \tau + \theta'\right)\right)$$

Local oscillator should match to phase θ.

Chapter 3:Electromechanical-Energy-Conversion

Chapter 3: Electromechanical-Energy-Conversion

Principles

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- The electromechanical-energy-conversion process takes place through the medium of the electric or magnetic field of the conversion device of which the structures depend on their respective functions.

- Transducers: microphone, pickup, sensor, loudspeaker
- Force producing devices: solenoid, relay, electromagnet
- Continuous energy conversion equipment: motor, generator

This chapter is devoted to the principles of electromechanical energy conversion and the analysis of the devices accomplishing this function. Emphasis is placed on the analysis of systems that use magnetic fields as the conversion medium.

- The concepts and techniques can be applied to a wide range of engineering situations involving electromechanical energy conversion.
- Based on the energy method, we are to develop expressions for forces and torques in magnetic-field-based electromechanical systems.

§3.1 Forces and Torques in Magnetic Field Systems

- The Lorentz Force Law gives the force F on a particle of charge q in the presence of electric and magnetic fields.

$F = q(E + v \times B)$ (3.1)

F : newtons, q : coulombs, E : volts/meter, B : telsas, v : meters/second

- In a pure electric-field system,

F   qE (3.2)

- In pure magnetic-field systems,

$F = q(v \times B)$ (3.3)

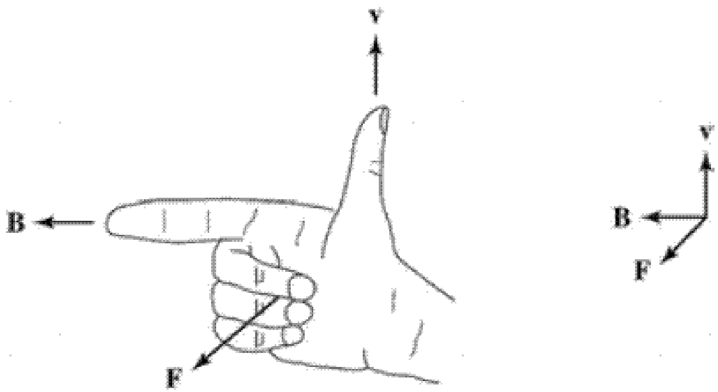


Figure 3.1 Right-hand rule for F=(q x v) B .

- For situations where large numbers of charged particles are in motion,

$F_v = \rho(E + v \times B)$ (3.4)

$J = \rho v$ (3.5)

$F_v = J \times B$ (3.6)

(charge density): coulombs/ m^3, F (force density): newtons/ m^3,

$J = \rho v$ (current density): amperes/ m^2.

- Most electromechanical-energy-conversion devices contain magnetic material.
- Forces act directly on the magnetic material of these devices which are constructed of rigid, nondeforming structures.
- The performance of these devices is typically determined by the net force, or torque, acting on the moving component. It is rarely necessary to calculate the details of the internal force distribution.
- Just as a compass needle tries to align with the earth’s magnetic field, the two sets of fields associated with the rotor and the stator of rotating machinery attempt to align, and torque is associated with their displacement from alignment.
  - In a motor, the stator magnetic field rotates ahead of that of the rotor, pulling on it and performing work.
  - For a generator, the rotor does the work on the stator.
    - The Energy Method
- Based on the principle of conservation of energy: energy is neither created nor destroyed; it is merely changed in form.
- Fig. 3.2(a): a magnetic-field-based electromechanical-energy-conversion device.

  - A lossless magnetic-energy-storage system with two terminals
  - The electric terminal has two terminal variables: e (voltage), i (current).
  - The mechanical terminal has two terminal variables: f_{fld}(force), x (position)
  - The loss mechanism is separated from the energy-storage mechanism.

–Electrical losses: ohmic losses…

–Mechanical losses: friction, windage…

Fig. 3.2(b): a simple force-producing device with a single coil forming the electric terminal, and a movable plunger serving as the mechanical terminal.

- The interaction between the electric and mechanical terminals, i.e. the electromechanical energy conversion, occurs through the medium of the magnetic stored energy.

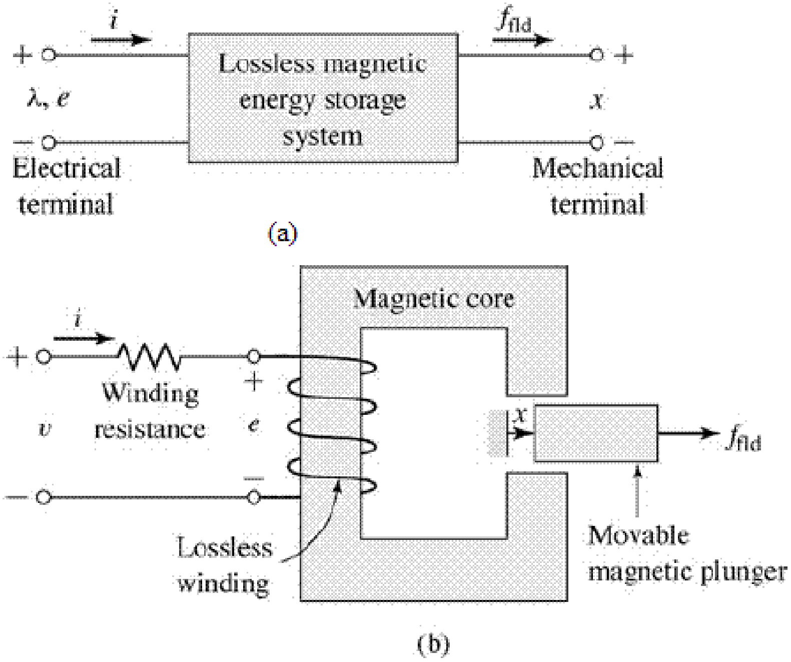


Figure 3.2(a) Schematic magnetic-field electromechanical-energy-conversion device;

(b) simple force-producing device.

- W_{fld}: the stored energy in the magnetic field

$\frac{\mathrm{dW}_{\mathrm{fld}}}{\mathrm{dt}} = \mathrm{ei} - f_{\mathrm{fld}}\frac{\mathrm{dx}}{\mathrm{dt}}$ (3.7)

$e = \frac{d\lambda}{\mathrm{dt}}$ (3.8)

$\mathrm{dW}_{\mathrm{fld}} = \mathrm{id}\lambda - f_{\mathrm{fld}}\mathrm{dx}$ (3.9)

- Equation (3.9) permits us to solve for the force simply as a function of the flux and the mechanical terminal position x .
- Equations (3.7) and (3.9) form the basis for the energy method.

§3.2 Energy Balance

- Consider the electromechanical systems whose predominant energy-storage mechanism is in magnetic fields. For motor action, we can account for the energy transfer as

$$\begin{array}{c} \text{Energy input} \\ \text{form electric} \\ \text{source} \end{array} = \begin{array}{c} \text{Mechanical} \\ \text{energy} \\ \text{ouput} \end{array} + \begin{array}{c} \text{Increase in energy} \\ \text{stored in magnetic} \\ \text{field} \end{array} + \begin{array}{c} \text{Energy} \\ \text{converted} \\ \text{into heat} \end{array} \quad (3.10)$$

- Note the generator action.

- The ability to identify a lossless-energy-storage system is the essence of the energy method.

- This is done mathematically as part of the modeling process.
- For the lossless magnetic-energy-storage system of Fig. 3.3(a), rearranging (3.9) in form of (3.10) gives

$\mathrm{dW_{elec}} = \mathrm{dW_{mech}} + \mathrm{dW_{fld}}$ (3.11)

where

$\mathrm{dW_{elec}} = \mathrm{id}\lambda$differential electric energy input

$\mathrm{dW_{mech}} = f_{\mathrm{fld}}\mathrm{dx}$differential mechanical energy output

$\mathrm{dW_{fld}}$differential change in magnetic stored energy

- Here e is the voltage induced in the electric terminals by the changing magnetic stored energy. It is through this reaction voltage that the external electric circuit supplies power to the coupling magnetic field and hence to the mechanical output terminals.

$\mathrm{dW_{elec}} = \mathrm{eidt}$ (3.12)

- The basic energy-conversion process is one involving the coupling field and its action and reaction on the electric and mechanical systems.
- Combining (3.11) and (3.12) results in

$\mathrm{dW_{elec}} = \mathrm{eidt} = \mathrm{dW_{mech}} + \mathrm{dW_{fld}}$ (3.13)

§3.3 Energy in Singly-Excited Magnetic Field Systems

- We are to deal energy-conversion systems: the magnetic circuits have air gaps between the stationary and moving members in which considerable energy is stored in the magnetic field.

- This field acts as the energy-conversion medium, and its energy is the reservoir between the electric and mechanical system.

- Fig. 3.3 shows an electromagnetic relay schematically. The predominant energy storage occurs in the air gap, and the properties of the magnetic circuit are determined by the dimensions of the air gap.

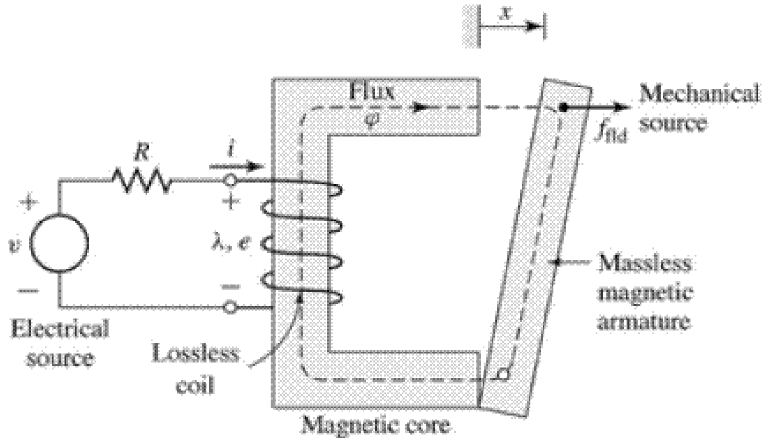


Figure 3.3Schematic of an electromagnetic relay.

$\lambda = L(x)I$(3.14)

$\mathrm{dW}_{\mathrm{mech}} = f_{\mathrm{fld}}\mathrm{dx}$(3.15)

$\mathrm{dW}_{\mathrm{fld}} = \mathrm{id}\lambda - f_{\mathrm{fld}}\mathrm{dx}$(3.16)

- W_{fld} is uniquely specified by the values of λ and x . Therefore, $\lambda$and x are referred to as state variables.
- Since the magnetic energy storage system is lossless, it is a conservative system. W_{fld} is the same regardless of how λ and x are brought to their final values. See Fig. 3.4 where tow separate paths are shown.

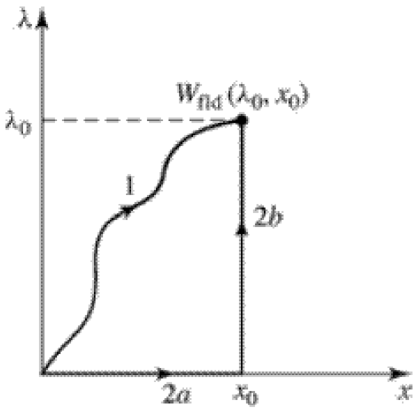


Figure 3.4Integration paths for Wfld

$W_{\mathrm{fld}}(\lambda_0,x_0) = \int_{\text{path 2a}} \mathrm{dW_{fld}} + \int_{\text{path 2b}} \mathrm{dW_{fld}}$ (3.17)

On path 2a, $d\lambda = 0$ and $f_{\mathrm{fld}} = 0$. Thus, $\mathrm{dW_{fld}} = 0$ on path 2a.

On path 2b, dx   0 .

Therefore, (3.17) reduces to the integral of i$\mathrm{d}\lambda$ over path 2b.

$W_{\mathrm{fld}}(\lambda_0,x_0) = \int_0^{\lambda_0} i(\lambda,x_0)d\lambda$ (3.18)

For a linear system in which λ is proportional to i , (3.18) gives

$W_{\mathrm{fld}}(\lambda,x) = \int_0^{\lambda} i(\lambda',x)d\lambda' = \int_0^{\lambda} \frac{\lambda'}{L(x)} d\lambda' = \frac{1}{2}\frac{\lambda^2}{L(x)}$ (3.19)

- V : the volume of the magnetic field

$W_{\mathrm{fld}} = \int_v (\int_0^B H.dB')\mathrm{dV}$ (3.20)

If $B = \mu H$,

$W_{\mathrm{fld}} = \int_v (\frac{B^2}{2\mu})\mathrm{dV}$ (3.21)

§3.4 Determination of Magnetic Force and Torque form Energy

- The magnetic stored energy W_{fld} is a state function, determined uniquely by the values of the independent state variables λ and x.

$\mathrm{dW_{fld}}(\lambda,x) = \mathrm{i}d\lambda - f_{\mathrm{fld}}\mathrm{dx}$ (3.22)

$$\mathrm{dF}(x_1,x_2) = \frac{\partial F}{\partial x_1}\ |_{x_2}\ \mathrm{dx}_1 + \frac{\partial F}{\partial x_2}\ |_{x_1}\ \mathrm{dx}_2 \text{ (3.23)}$$

$$\mathrm{dW}_{\mathrm{fld}}(\lambda,x) = \frac{\partial W_{\mathrm{fld}}}{\partial \lambda}\ |_x\ d\lambda + \frac{\partial W_{\mathrm{fld}}}{\partial x}\ |_\lambda\ \mathrm{dx} \text{ (3.24)}$$

Comparing (3.22) with (3.24) gives (3.25) and (3.26):

$$i = \frac{\partial W_{\mathrm{fld}}(\lambda,x)}{\partial \lambda}\ |_x \text{ (3.25)}$$

$$f_{\mathrm{fld}} = -\frac{\partial W_{\mathrm{fld}}(\lambda,x)}{\partial x}\ |_\lambda \text{(3.26)}$$

- Once we know W_{fld} as a function of λ and as a function of λ and i(λ, x) .
- Equation (3.26) can be used to solve for the mechanical force $f_{\mathrm{fld}}(\lambda,x)$.The partial derivative is taken while holding the flux linkages λ constant.
- For linear magnetic systems for which $\lambda = L(x)i$, the force can be found as

$$f_{\mathrm{fld}} = -\frac{\partial}{\partial x}\ \frac{1}{2}\ \frac{\lambda^2}{L(x)}\ \ |_\lambda = \frac{\lambda^2}{2\mathrm{L}(x)^2}\frac{\mathrm{dL}(x)}{\mathrm{dx}} \text{ (3.27)}$$

$$f_{\mathrm{fld}} = \frac{i^2}{2}\frac{\mathrm{dL}(x)}{\mathrm{dx}} \text{ (3.28)}$$

- For a system with a rotating mechanical terminal, the mechanical terminal variables become the angular displacement θ and the torque T_{fld} .

$$\mathrm{dW}_{\mathrm{fld}}(\lambda,\theta) = \mathrm{id}\lambda - T_{\mathrm{fld}}d\theta \text{ (3.29)}$$

$$T_{\mathrm{fld}} = -\frac{\partial W_{\mathrm{fld}}(\lambda,\theta)}{\partial \theta}\ |_\lambda \text{ (3.30)}$$

- For linear magnetic systems for which $\lambda = L(\theta)i$:

$$W_{\mathrm{fld}}(\lambda,\theta) = \frac{1}{2}\ \frac{\lambda^2}{L(\theta)} \text{ (3.31)}$$

$$T_{\mathrm{fld}} = -\frac{\partial}{\partial \theta}\ \frac{1}{2}\ \frac{\lambda^2}{L(\theta)}\ \ |_\lambda = \frac{1}{2}\ \frac{\lambda^2}{L(\theta)^2}\frac{\mathrm{dL}(\theta)}{d\theta} \text{ (3.32)}$$

$$T_{\mathrm{fld}} = \frac{i^2}{2}\frac{\mathrm{dL}(\theta)}{d\theta} \text{ (3.33)}$$

§3.5 Determination of Magnetic Force and Torque from Coenergy

- Recall that in §3.4, the magnetic stored energy W_{fld} is a state function, determined uniquely by the values of the independent state variables λ and x .

$$\mathrm{dW}_{\mathrm{fld}}(\lambda,x) = \mathrm{id}\lambda - f_{\mathrm{fld}}\mathrm{dx} \text{ (3.34)}$$

$$\mathrm{dW}_{\mathrm{fld}}(\lambda,x) = \frac{\partial W_{\mathrm{fld}}}{\partial \lambda}\ |_x\ d\lambda + \frac{\partial W_{\mathrm{fld}}}{\partial x}\ |_\lambda\ \mathrm{dx} \text{ (3.35)}$$

$i = \frac{\partial W_{\text{fld}}(\lambda,x)}{\partial \lambda}\ |_{x}$ (3.36)

$f_{\text{fld}} = \frac{\partial W_{\text{fld}}(\lambda,x)}{\partial x}\ |_{\lambda}$ (3.37)

- Coenergy: from which the force can be obtained directly as a function of the current.The selection of energy or coenergy as the state function is purely a matter of convenience.

- The coenergy $W_{\text{fld}}^{'}(i,x)$ is defined as a function of i and x such that

$W_{\text{fld}}^{'}(i,x) = i\lambda - W_{\text{fld}}(\lambda,x)$ (3.38)

$d(i\lambda) = \text{id}\lambda = \lambda\text{di}$ (3.39)

$dW_{\text{fld}}^{'}(i,x) = d(i\lambda) - \text{dW}_{\text{fld}}(\lambda,x)$ (3.40)

$dW_{\text{fld}}^{'}(i,x) = \lambda\text{di} + f_{\text{fld}}\text{dx}$ (3.41)

- From (3.37), the coenergy $W_{\text{fld}}^{'}(i,x)$can be seen to be a state function of the two independent variables i and x .

$dW_{\text{fld}}^{'}(i,x) = \frac{\partial W_{\text{fld}}^{'}}{\partial i}\ |_{x}\ \text{di} + \frac{\partial W_{\text{fld}}^{'}}{\partial x}\ |_{i}\ \text{dx}$ (3.42)

$\lambda = \frac{\partial W_{\text{fld}}^{'}(i,x)}{\partial i}\ |_{x}$ (3.43)

$f_{\text{fld}} = \frac{\partial W_{\text{fld}}^{'}(i,x)}{\partial x}\ |_{i}$ (3.44)

- For any given system, (3.26) and (3.40) will give the same result.
- By analogy to (3.18) in §3.3, the coenergy can be found as (3.41)

$W_{\text{fld}}(\lambda_0,x_0) = \int_0^{\lambda_0} i(\lambda,x_0)d\lambda$ (3.42)

$W_{\text{fld}}^{'}(i,x) = \int \lambda(i^{'},x)di^{'}$ (3.43)

For linear magnetic systems for which $\lambda = L(x)i$,

$W_{\text{fld}}^{'}(i,x) = \frac{1}{2}L(x)i^2$ (3.44)

$f_{\text{fld}} = \frac{i^2}{2}\frac{\text{dL}(x)}{\text{dx}}$ (3.45)

- For a system with a rotating mechanical displacement,

$$W_{\mathrm{fld}}^{'}(i,\theta) = \int_0^i \lambda(i^{'},\theta)di^{'} \text{ (3.46)}$$

$$T_{\mathrm{fld}} = \frac{\partial W_{\mathrm{fld}}^{'}(i,\theta)}{\partial\theta}\ |_i \text{ (3.47)}$$

If the system is magnetically linear,

$$W_{\mathrm{fld}}^{'}(i,\theta) = \frac{1}{2}L(\theta)i^2 \text{ (3.48)}$$

$$T_{\mathrm{fld}} = \frac{i^2}{2}\frac{\mathrm{dL}(\theta)}{d\theta} \text{ (3.49)}$$

(3.47) is identical to the expression given by (3.33).

- In field-theory terms, for soft magnetic materials

$$W_{\mathrm{fld}}^{'} = \int_V \int_0^{H_0} \mathrm{BdH\ dV} \text{ (3.50)}$$

$$W_{\mathrm{fld}}^{'} = \int_v \frac{\mu H^2}{2}\mathrm{dV} \text{ (3.51)}$$

For permanent-magnet (hard) materials

$$W_{\mathrm{fld}}^{'} = \int_v \int_{H_c}^{H_0} \mathrm{BdH\ dV} \text{ (3.52)}$$

- For a magnetically-linear system, the energy and coenergy (densities) are numerically equal: $\lambda^2/2\mathrm{L} = \frac{1}{2}\mathrm{Li}^2, B^2/2\mu = \frac{1}{2}\mu H^2$. For a nonlinear system in which λ and i or B and H are not linearly proportional, the two functions are not even numerically equal.

$$W_{\mathrm{fld}} + W_{\mathrm{fld}}^{'} = \lambda i \text{ (3.53)}$$

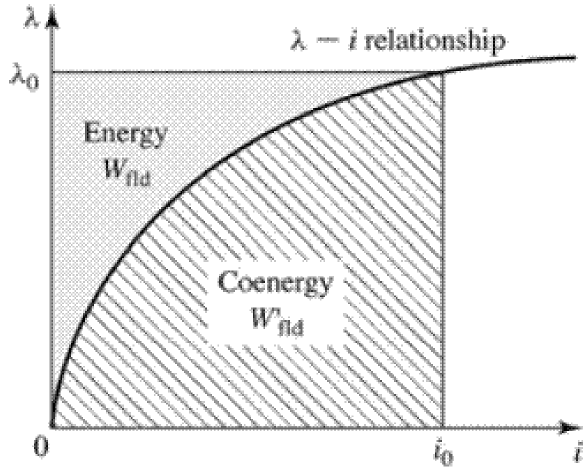


Figure 3.5Graphical interpretation of energy and coenergy in a singly-excited system.

- Consider the relay in Fig. 3.3. Assume the relay armature is at position x so that the device operating at point a in Fig. 3.6. Note that

$$f_{\mathrm{fld}} = -\frac{\partial W_{\mathrm{fld}}(\lambda,x)}{\partial x}\ |_{\lambda} \simeq \lim_{\Delta x\to 0}\frac{-\Delta W_{\mathrm{fld}}}{\Delta x}\ |_{\lambda}\ \text{and} f_{\mathrm{fld}} = \frac{\partial W^{'}_{\mathrm{fld}}(i,x)}{\partial x}\ |_{i} \simeq \lim_{\Delta x\to 0}\frac{\Delta W^{'}_{\mathrm{fld}}}{\Delta x}\ |_{i}$$

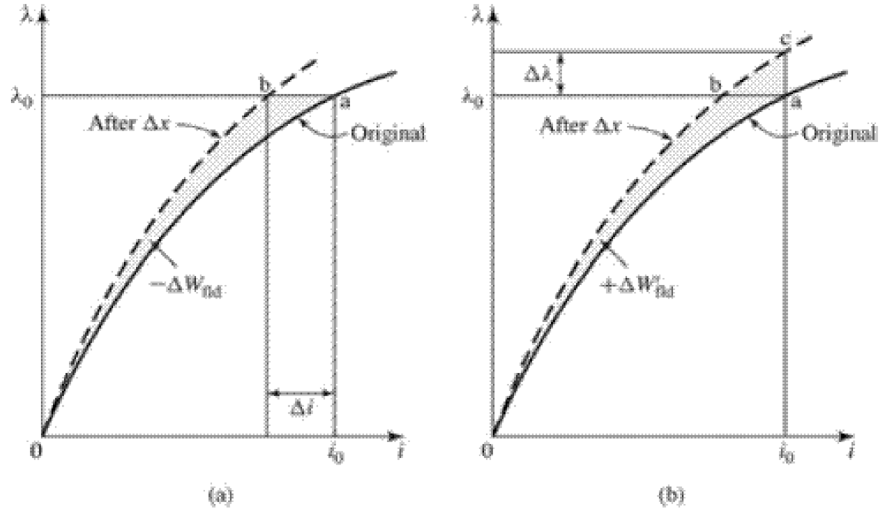


Figure 3.6Effect of Δx on the energy and coenergy of a singly-excited device:

(a) change of energy with λ held constant; (b) change of coenergy with i held constant.

- The force acts in a direction to decrease the magnetic field stored energy at constant flux or to increase the coenergy at constant current.
- In a singly-excited device, the force acts to increase the inductance by pulling on members so as to reduce the reluctance of the magnetic path linking the winding.

§3.6 Multiply-Excited Magnetic Field Systems

- Many electromechanical devices have multiple electrical terminals.

- Measurement systems: torque proportional to two electric signals; power as the product of voltage and current.
- Energy conversion devices: multiply-excited magnetic field system.
- A simple system with two electrical terminals and one mechanical terminal: Fig.3.7.

    - Three independent variables: $\{\theta,\lambda_1,\lambda_2\}$, $\{\theta,i_1,i_2\}$, $\{\theta,\lambda_1,i_2\}$, or $\{\theta,i_1,\lambda_2\}$

$\mathrm{dW}_{\mathrm{fld}}(\lambda_1,\lambda_2,\theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - T_{\mathrm{fld}} d\theta$ (3.54)

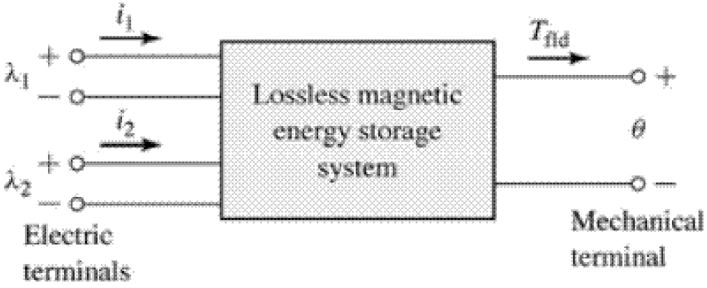


Figure 3.7Multiply-excited magnetic energy storage system.

$i_1 = \frac{\partial W_{\mathrm{fld}}(\lambda_1,\lambda_2,\theta)}{\partial \lambda_1} \,|_{\lambda_2,\theta}$ (3.55)

$i_2 = \frac{\partial W_{\mathrm{fld}}(\lambda_1,\lambda_2,\theta)}{\partial \lambda_2} \,|_{\lambda_1,\theta}$(3.56)

$T_{\mathrm{fld}} = -\frac{\partial W_{\mathrm{fld}}(\lambda_1,\lambda_2,\theta)}{\partial \theta} \,|_{\lambda_1,\lambda_2}$ (3.57)

To find W_{fld} , use the path of integration in Fig. 3.14.

$W_{\mathrm{fld}}(\lambda_{1_0},\lambda_{2_0},\theta_0) = \int_0^{\lambda_{2_0}} i_2(\lambda_1 = 0,\lambda_2,\theta = \theta_0) d\lambda_2 + \int_0^{\lambda_{1_0}} i_1(\lambda_1,\lambda_2 = \lambda_{2_0},\theta = \theta_0) d\lambda_1$ (3.58)

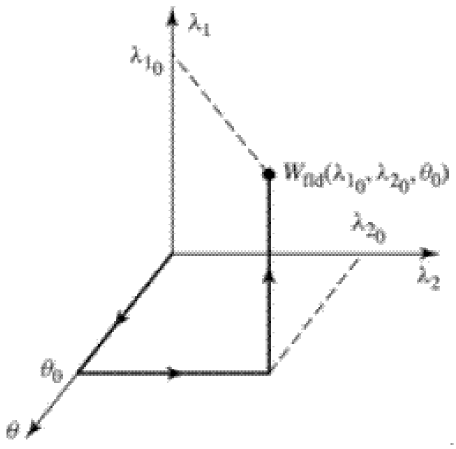


Figure 3.8Integration path to obtain $W_{\text{fld}}(\lambda_{1_0},\lambda_{2_0},\theta_0)$.

- In a magnetically-linear system,

$\lambda_1 = L_{11} i_1 + L_{12} i_2$ (3.59)

$\lambda_2 = L_{21} i_1 + L_{22} i_2$ (3.60)

$L_{12} = L_{21}$ (3.61)

Note that $L_{\text{ij}} = L_{\text{ij}}(\theta)$

$i_1 = \frac{L_{22}\lambda_1 - L_{12}\lambda_2}{D}$ (3.62)

$i_2 = \frac{-L_{21}\lambda_1 + L_{11}\lambda_2}{D}$ (3.63)

$D = L_{11}L_{22} - L_{12}L_{21}$ (3.64)

The energy for this linear system is

$$\begin{aligned} W_{\text{fld}}(\lambda_{1_0},\lambda_{2_0},\theta_0) &= \int_0^{\lambda_{2_0}} \frac{L_{11}(\theta_0)\lambda_2}{D(\theta_0)} d\lambda_2 + \int \frac{(L_{22}(\theta_0)\lambda_1 - L_{12}(\theta_0)\lambda_{2_0})}{D(\theta_0)} \\ &= \frac{1}{2\text{D}(\theta_0)} L_{11}(\theta_0)\lambda_{2_0}^2 + \frac{1}{2\text{D}(\theta_0)} L_{22}(\theta_0)\lambda_{1_0}^2 - \frac{L_{12}(\theta_0)}{D(\theta_0)} \lambda_{1_0}\lambda_{2_0} \end{aligned} \quad (3.65)$$

- Coenergy function for a system with two windings can be defined as:

$W^{'}_{\text{fld}}(i_1,i_2,\theta) = \lambda_1 i_1 + \lambda_2 i_2 - W_{\text{fld}}$ (3.66)

$dW^{'}_{\text{fld}}(i_1,i_2,\theta) = \lambda_1 \text{di}_1 + \lambda_2 \text{di}_2 + T_{\text{fld}} d\theta$ (3.67)

$\lambda_1 = \frac{\partial W_{\text{fld}}(i_1,i_2,\theta)}{\partial i_1} \ |_{i_2,\theta}$ (3.68)

$\lambda_2 = \frac{\partial W^{'}_{\text{fld}}(i_1,i_2,\theta)}{\partial i_2}\ |_{i_1,i_2}$ (3.69)

$T_{\text{fld}} = \frac{\partial W^{'}_{\text{fld}}(i_1,i_2,\theta)}{\partial \theta}\ |_{i_1,i_2}$ (3.70)

$W^{'}_{\text{fld}}(i_1,i_2,\theta_0) = \int_0^{i_{2_0}} \lambda_2(i_1 = 0,i_2,\theta = \theta_0)\text{di}_2 + \int_0^{\lambda_{1_0}} \lambda_1(i_1,i_2 = i_{2_0},\theta = \theta_0)\text{di}_1$ (3.71)

- For the linear system:

$W^{'}(i_1,i_2,\theta_0) = \frac{1}{2}L_{11}(\theta)i_1^2 + \frac{1}{2}L_{22}(\theta)i_2^2 + L_{12}(\theta)i_1 i_2$ (3.72)

$T_{\text{fld}} = \frac{\partial W^{'}_{\text{fld}}(i_1,i_2,\theta_0)}{\partial \theta}\ |_{i_1,i_2} = \frac{i_1^2}{2}\frac{\text{dL}_{11}(\theta)}{d\theta} + \frac{i_2^2}{2}\frac{\text{dL}_{22}(\theta)}{d\theta} + i_1 i_2 \frac{\text{dL}_{12}(\theta)}{d\theta}$ (3.73)

- Note that (3.70) is simpler than (3.57). That is, the coenergy function is a relatively simple function of displacement.
- The use of a coenergy function of the terminal currents simplifies the determination of torque or force.
- Systems with more than two electrical terminals are handled in analogous fashion.
    - System with linear displacement:

$W_{\text{fld}}(\lambda_{1_0},\lambda_{2_0},x_0) = \int_0^{\lambda_{2_0}} i_2(\lambda_1 = 0,\lambda_2,x = x_0)d\lambda_2 + \int_0^{\lambda_{1_0}} i_1(\lambda_1,\lambda_2 = \lambda_{2_0},x = x_0)d\lambda_1$ (3.74)

$W^{'}_{\text{fld}}(i_{1_0},i_{2_0},x_0) = \int_0^{\lambda_{2_0}} \lambda_2(i_1,i_2,x = x_0)\text{di}_2 + \int_0^{\lambda_{1_0}} \lambda_1(i_1,i_2 = i_{2_0},x = x_0)\text{di}_1$

$f_{\text{fld}} = -\frac{\partial W_{\text{fld}}(\lambda_1,\lambda_2,x)}{\partial x}\ |_{\lambda_1,\lambda_2}$ (3.76)

$f_{\text{fld}} = -\frac{\partial W^{'}_{\text{fld}}(i_1,i_2,x)}{\partial x}\ |_{i_1,i_2}$ (3.77)

For a magnetically-linear system,

$W^{'}_{\text{fld}}(i_1,i_2,x) = \frac{1}{2}L_{11}(x)i_1^2 + L_{22}(x)i_2^2 + L_{12}(x)i_1 i_2$ (3.78)

$f_{\text{fld}} = \frac{i_1^2}{2}\frac{\text{dL}_{11}(x)}{\text{dx}} + \frac{i_2^2}{2}\frac{\text{dL}_{22}(x)}{\text{dx}} + i_1 i_2 \frac{\text{dL}_{12}(x)}{\text{dx}}$ (3.79)

Chapter 4: Introduction to Rotating Machines

Chapter 4: Introduction to Rotating Machines

This lecture note is based on the textbook # 1. Electric Machinery - A.E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans- 6th edition- Mc Graw Hill series in Electrical Engineering. Power and Energy

- The objective of this chapter is to introduce and discuss some of the principles underlying the performance of electric machinery, both ac and dc machines.

§4.1 Elementary Concepts

- Voltages can be induced by time-varying magnetic fields. In rotating machines, voltages are generated in windings or groups of coils by rotating these windings mechanically through a magnetic field, by mechanically rotating a magnetic field past the winding, or by designing the magnetic circuit so that the reluctance varies with rotation of the rotor.

- The flux linking a specific coil is changed cyclically, and a time-varying voltage is generated.
- Electromagnetic energy conversion occurs when changes in the flux linkage result from mechanical motion.
- A set of such coils connected together is typically referred to as an armature winding, a winding or a set of windings carrying ac currents.
  - In ac machines such as synchronous or induction machines, the armature winding is typically on the stator. (the stator winding)
  - In dc machines, the armature winding is found on the rotor. (the rotor winding)

- Synchronous and dc machines typically include a second winding (or set of windings), referred to as the field winding, which carrys dc current and which are used to produce the main operating flux in the machine.
  - In dc machines, the field winding is found on the stator.
  - In synchronous machines, the field winding is found on the rotor.
  - Permanent magnets can be used in the place of field windings.

- In most rotating machines, the stator and rotor are made of electrical steel, and the windings are installed in slots on these structures.The stator and rotor structures are typically built from thin laminations of electrical steel, insulated from each other, to reduce eddy-current losses.

§4.2 Introduction to AC And DC Machines

§4.2.1 AC Machines

- Traditional ac machines fall into one of two categories: synchronous and induction.

- In synchronous machines, rotor-winding currents are supplied directly from the stationary frame through a rotating contact.
- In induction machines, rotor currents are induced in the rotor windings by a combination of the time-variation of the stator currents and the motion of the rotor relative to the stator.

- Synchronous Machines

Fig. 4.1: a simplified salient-pole ac synchronous generator with two poles.

- The armature winding is on the stator, and the field winding is on the rotor.
- The field winding is excited by direct current conducted to it by means of stationary carbon brushes that contact rotating slip rings or collector rings.
- It is advantages to have the single, low-power field winding on the rotor while having the high-power, typically multiple-phase, armature winding on the stator.
- Armature winding (a, a) consists of a single coil of N turns.
- Conductors forming these coil sides are connected in series by end connections.
- The rotor is turned at a constant speed by a source of mechanical power connected to its shaft. Flux paths are shown schematically by dashed lines.

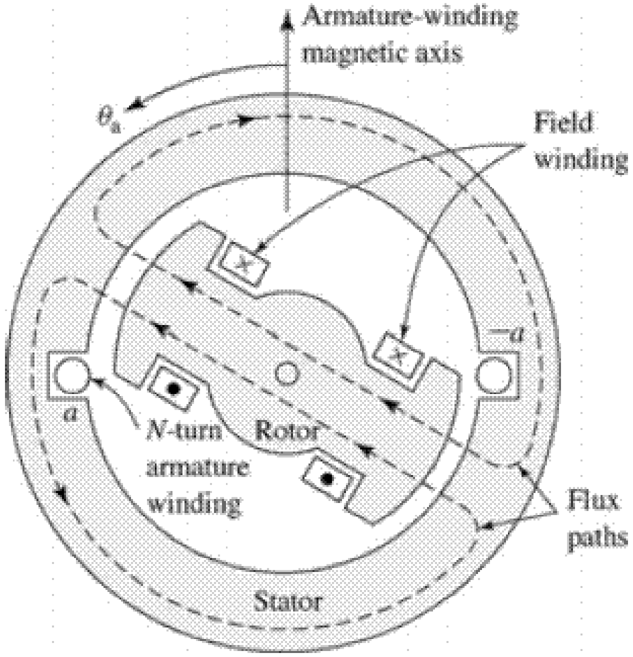


Figure 4.1 Schematic view of a simple, two-pole, single-phase synchronous generator.

Assume a sinusoidal distribution of magnetic flux in the air gap of the machine in Fig.4.1.

- The radial distribution of air-gap flux density B is shown in Fig. 4.2(a) as a function of the spatial angle θ around the rotor periphery.
- As the rotor rotates, the flux –linkages of the armature winding change with time and the resulting coil voltage will be sinusoidal in time as shown in Fig 4.2(b). The frequency in cycles per second (Hz) is the same as the speed of the rotor in revolutions in second (rps).
- A two-pole synchronous machine must revolve at 3600 rpm to produce a 60Hz voltage.
- Note the terms “rpm” and “rps”.

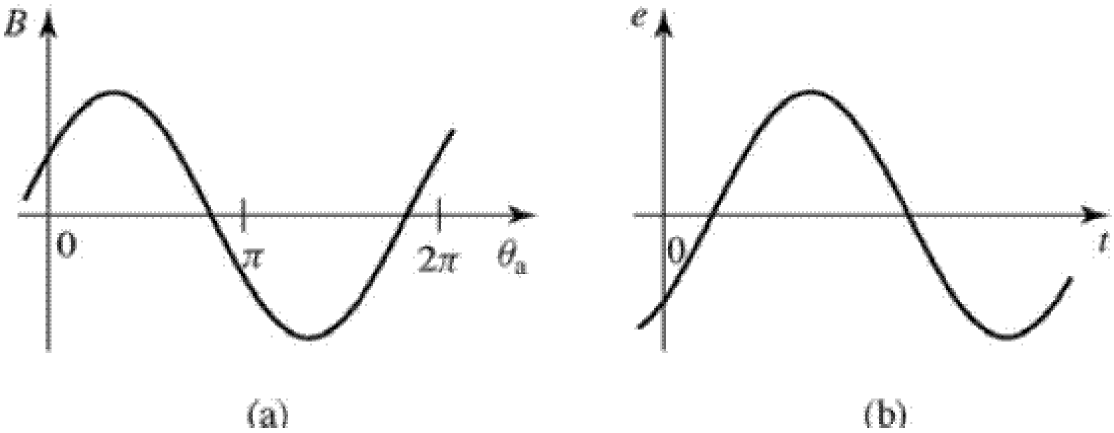


Figure 4.2 (a) Space distribution of flux density and (b) corresponding waveform of

the generated voltage for the single-phase generator of Fig. 4.1.

A great many synchronous machines have more than two poles. Fig 4.3 shows in schematic form a four-pole single-phase generator.

- The field coils are connected so that the poles are of alternate polarity.
- The armature winding consists of two coils $(a_1, -a_1)$and $(a_2, -a_2)$ connected in series by their end connections.
- There are two complete wavelengths, or cycles, in the flux distribution around the periphery, as shown in Fig. 4.4.
- The generated voltage goes through two complete cycles per revolution of the rotor.
- The frequency in Hz is thus twice the speed in rps.

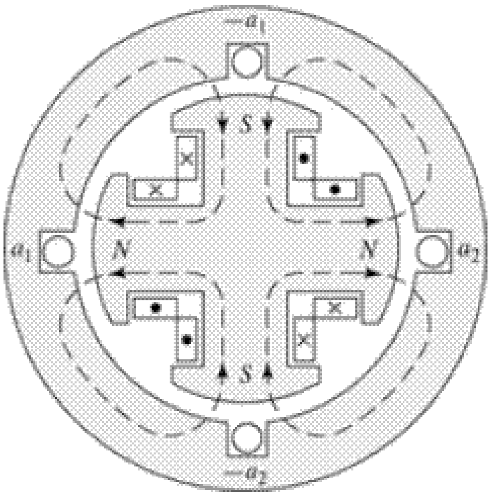


Figure 4.3 Schematic view of a simple, four-pole, single-phase synchronous generator.

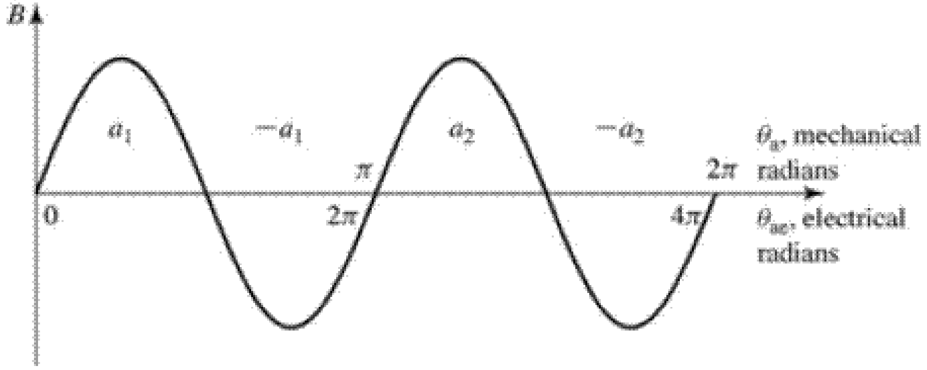


Figure 4.4 Space distribution of the air-gap flux density in an idealized,

four-pole synchronous generator.

When a machine has more than two poles, it is convenient to concentrate on a single pair of poles and to express angles in electrical degrees or electrical radians rather than in physical units.

- One pair of poles equals 360 electrical degrees or 2 π electrical radians.
- Since there are poles/2 wavelengths, or cycles, in one revolution, it follows that

$$\theta_{\mathrm{ae}} = \left(\frac{\text{poles}}{2}\right)\theta_a \quad (4.1)$$

Where $\theta_{\mathrm{ae}}$is the angle in electrical units and θ_a is the spatial angle.

- The coil voltage of a multipole machine passes through a complete cycle every time a pair of poles sweeps by, or (poles/2) times each revolution. The electrical frequency f_e of the voltage generated is therefore

$$f_e = \left(\frac{\text{poles}}{2}\right)\frac{n}{60}\,\text{Hz} \quad (4.2)$$

where n is the mechanical speed in rpm.Note that $\omega_e = (\text{poles}/2)\omega_m$

-
  - The rotors shown in Figs.4.1 and 4.3 have salient, or projecting, poles with concentrated windings. Fig.4.5 shows diagrammatically a nonsalient-pole, or cylindrical, rotor.

- The field winding is a two-pole distributed winding; the coil sides are distributed in multiple slots around the rotor periphery and arranged to produce an approximately sinusoidal distribution of radial air-gap flux.
- Most power systems in the world operate at frequencies of either 50 or 60 Hz.
- A salient-pole construction is characteristic of hydroelectric generators because hydraulic turbines operate at relatively low speeds, and hence a relatively large number of poles is required to produce the desired frequency.
- Steam turbines and gas turbines operate best at relatively high speeds, and turbine- driven alternators or turbine generators are commonly two- or four-pole cylindrical- rotor machines.

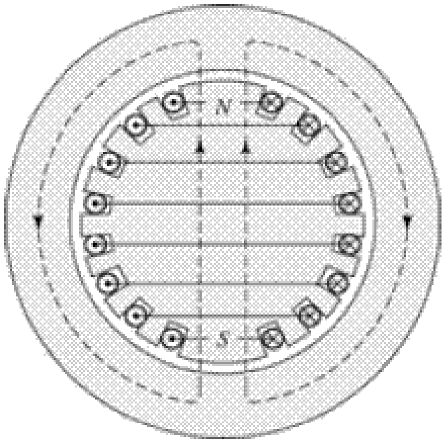
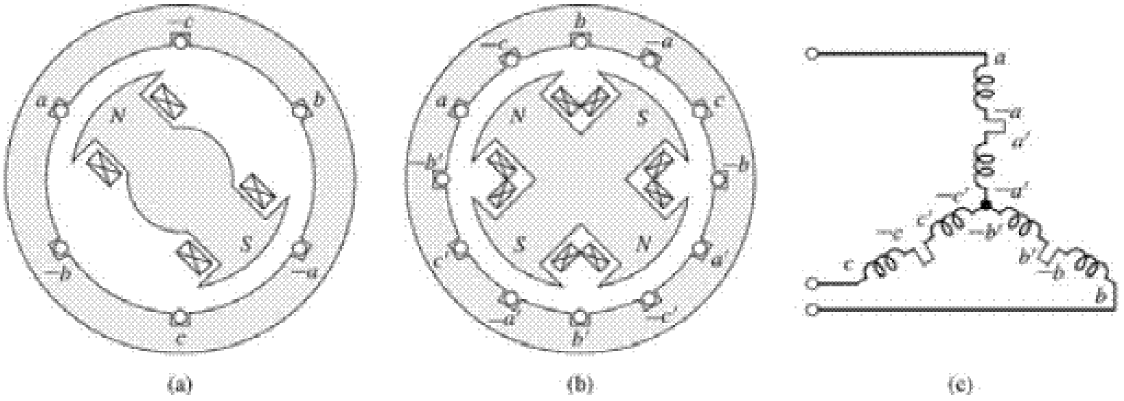


Figure 4.5 Elementary two-pole cylindrical-rotor field winding.

Most of the world’s power systems are three-phase systems. With very few exceptions, synchronous generators are three-phase machines.

- A simplified schematic view of a three-phase, two-pole machine with one coil per phase is shown in Fig. 4.6 (a)
- Fig. 4.6(b) depicts a simplified three-phase, four-pole machine. Note that a minimum of two sets of coils must be used. In an elementary multipole machine, the minimum number of coils sets is given by one half the number of poles.
- Note that coils (a, a) and $(a', -a')$ can be connected in series or in parallel. Then the coils of the three phases may then be either Y- or Δ-connected. See Fig. 4.6(c).



(a) (b) (c)

Figure 4.6 Schematic views of three-phase generators: (a) two-pole, (b) four-pole, and

(c) Y connection of the windings.

- The electromechanical torque is the mechanism through which a synchronous generator converts mechanical to electric energy.
  - When a synchronous generator supplies electric power to a load, the armature current creates a magnetic flux wave in the air gap that rotates at synchronous speed.
  - This flux reacts with the flux created by the field current, and an electromechanical torque results from the tendency of these two magnetic fields to align.
  - In a generator this torque opposes rotation, and mechanical torque must be applied from the prime mover to sustain rotation.

- The counterpart of the synchronous generator is the synchronous motor.
    - Ac current supplied to the armature winding on the stator, and dc excitation is supplied to the field winding on the rotor. The magnetic field produced by the armature currents rotates at synchronous speed.
    - To produce a steady electromechanical torque, the magnetic fields of the stator and rotor must be constant in amplitude and stationary with respect to each other.
    - In a motor the electromechanical torque is in the direction of rotation and balances the opposing torque required to drive the mechanical load.
    - In both generators and motors, an electromechanical torque and a rotational voltage are produced which are the essential phenomena for electromechanical energy conversion.
    - Note that the flux produced by currents in the armature of a synchronous motor rotates ahead of that produced by the field, thus pulling on the field (and hence on the rotor) and doing work. This is the opposite of the situation in a synchronous generator, where the field does work as its flux pulls on that of the armature, which is lagging behind.

- Induction Machines

- Alternating currents are applied directly to the stator windings. Rotors currents are then produced by induction, i.e., transformer action.
    - Alternating currents flow in the rotor windings of an induction machine, in contrast to a synchronous machine in which a field winding on the rotor is excited with dc current.
    - The induction machine may be regarded as a generalized transformer in which electric power is transformed between rotor and stator together with a change of frequency and a flow of mechanical power.

- The induction motor is the most common of all motors.
    - The induction machine is seldom used as a generator.
    - In recent years it has been found to be well suited for wind-power applications.
    - It may also be used as a frequency changer.

- In the induction motor, the stator windings are essentially the same as those of a synchronous machine.The rotor windings are electrically short-circuited.
    - The rotor windings frequently have no external connections.
    - Currents are induced by transformer action from the stator winding.
    - Squirrel-cage induction motor: relatively expensive and highly reliable.
- The armature flux in the induction motor leads that of the rotor and produces an electromechanical torque.
    - The rotor does not rotate synchronously.
    - It is the slipping of the rotor with respect to the synchronous armature flux that gives rise to the induced rotor currents and hence the torque.
    - Induction motors operate at speeds less than the synchronous mechanical speed.
    - A typical speed-torque characteristic for an induction motor is shown in Fig.4.7.

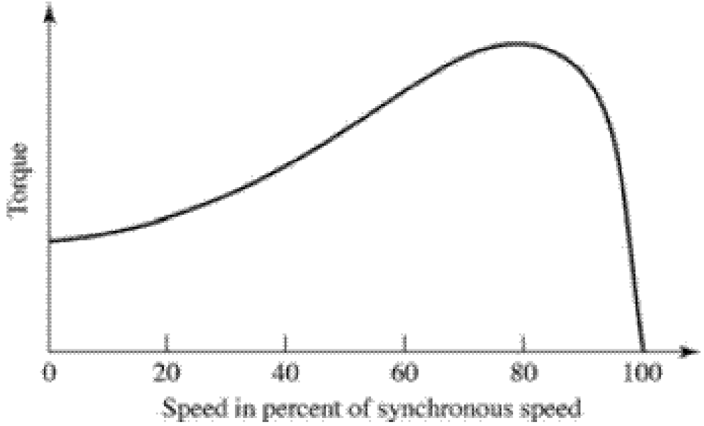


Figure 4.7 Typical induction-motor speed-torque characteristic.

§4.2.2 DC Machines

- DC Machines
- There are two sets of windings in a dc machine.

-
  - The armature winding is on the rotor with current conducted from it by means of carbon brushes.
  - The field winding is on the stator and is excited by direct current.

- An elementary two-pole dc generator is shown in Fig. 4.8.

- Armature winding: (a, a) , pitch 180^o
- The rotor is normally turned at a constant speed by a source of mechanical power connected the shaft.

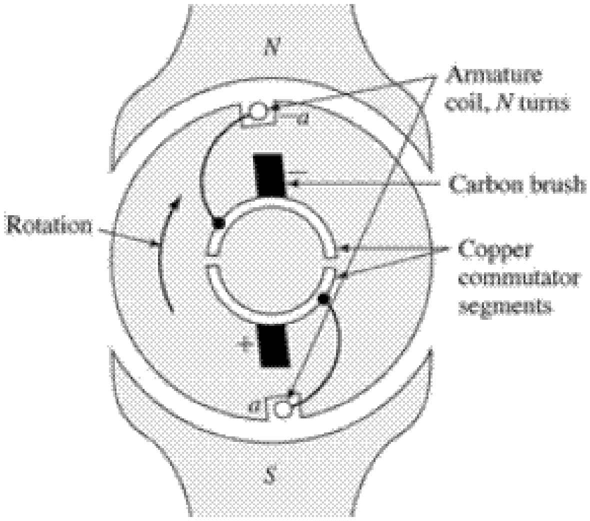


Figure 4.8 Elementary dc machine with commutator.

- The air-gap flux distribution usually approximates a flat-topped wave, rather than the sine wave found in ac machines, and is shown in Fig. 4.9(a).
- Rotation of the coil generates a coil voltage which is a time function having the same waveform as the spatial flux-density distribution.
- The voltage induced in an individual armature coil is an alternating voltage and rectification is produced mechanically by means of a commutator. Stationary carbon brushes held against the commutator surface connect the winding to the external armature terminal.
- The need for commutation is the reason why the armature windings are placed on the rotor.
- The commutator provides full-wave rectification, and the voltage waveform between brushes is shown in Fig. 4.9(b).

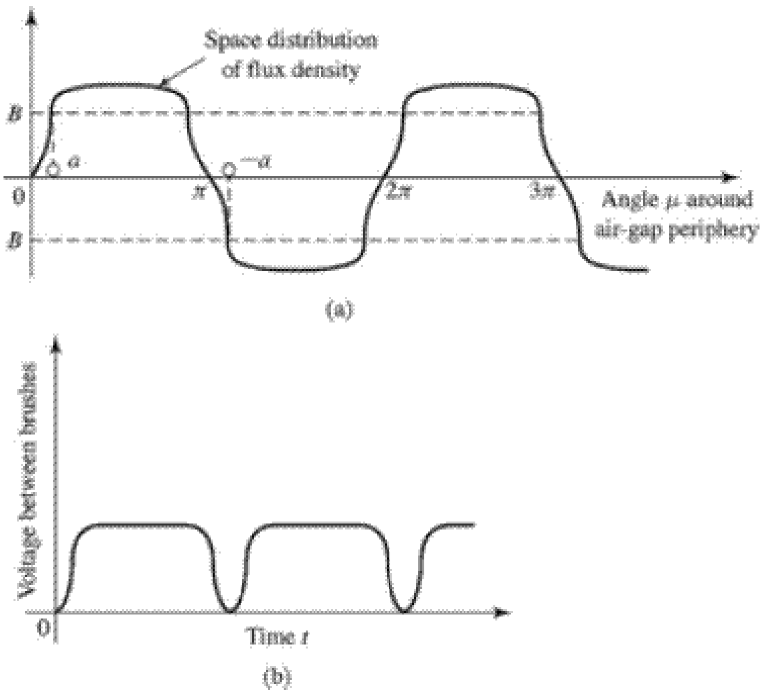


Figure 4.9 (a) Space distribution of air-gap flux density in an elementary dc machine;

(b) waveform of voltage between brushes.

It is the interaction of the two flux distributions created by the direct currents in the field and the armature windings that creates an electromechanical torque.

- If the machine is acting as a generator, the torque opposes rotation.
- If the machine is acting as a motor, the torque acts in the direction of the rotation.

§4.3 MMF of Distributed Windings

- Most armatures have distributed windings, i.e. windings which are spread over a number of slots around the air-gap periphery.

- The individual coils are interconnected so that the result is a magnetic field having the same number of poles as the field winding.
- Consider Fig. 4.10(a).
    - Full-pitch coil: a coil which spans 180 electrical degrees.

  - In Fig. 4.10(b), the air gap and winding are in developed form (laid out flat) and the air-gap mmf distribution is shown by the steplike distribution of amplitude

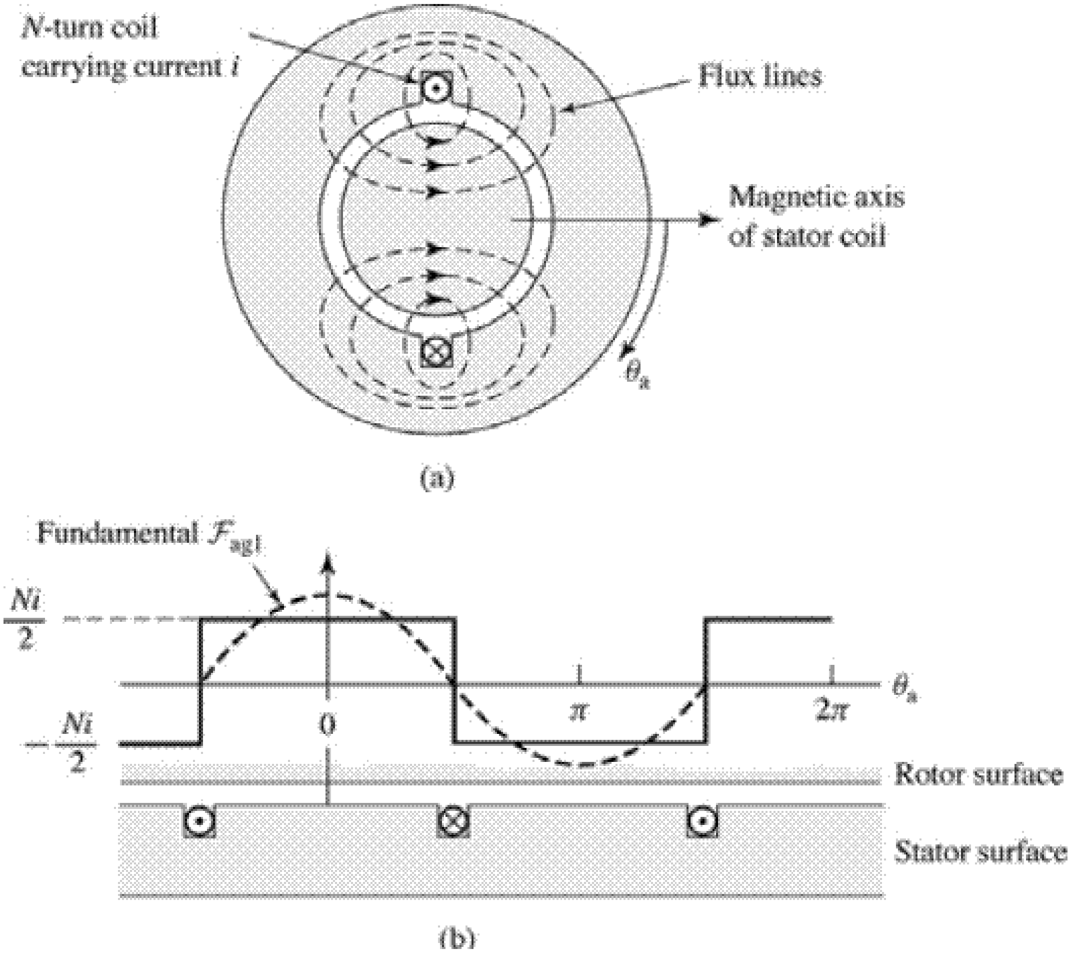


Figure 4.10 (a) Schematic view of flux produced by a concentrated, full-pitch winding in a machine with a uniform air gap. (b) The air-gap mmf produced by current in this winding.

§4.3.1 AC Machines

- It is appropriate to focus our attention on the space-fundamental sinusoidal component of the air-gap mmf.

- In the design of ac machines, serious efforts are made to distribute the coils making up the windings so as to minimize the higher-order harmonic components.

- The rectangular air-gap mmf wave of the concentrated two-pole, full-pitch coil of Fig.4.10(b) can be resolved to a Fourier series comprising a fundamental component and a series of odd harmonics.

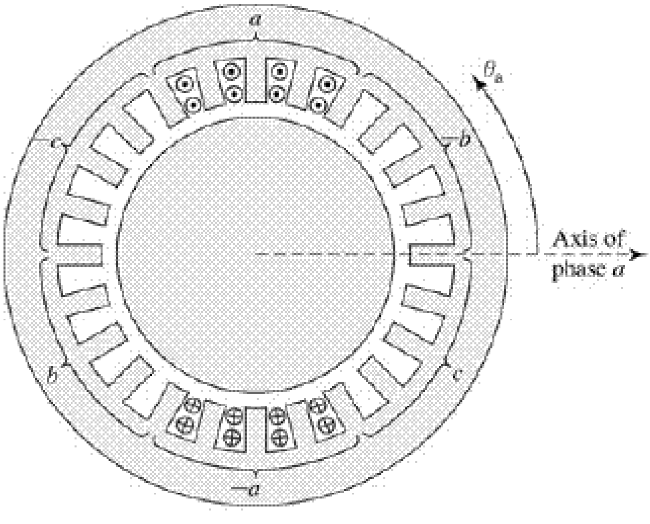
- The fundamental component $F_{\text{agl}}$and its amplitude $(F_{\text{agl}})_{\text{peak}}$are

$$F_{\text{agl}} = \frac{4}{\pi}\left(\frac{\text{Ni}}{2}\right)\cos\theta_a \text{ (4.3)}$$

$$(F_{\text{agl}})_{\text{peak}} = \frac{4}{\pi}\left(\frac{\text{Ni}}{2}\right) \text{ (4.4)}$$

Consider a distributed winding, consisting of coils distributed in several slots.

- Fig. 4.11(a) shows phase a of the armature winding of a simplified two-pole, three-phase ac machine and phases b and c occupy the empty slots.
- The windings of the three phases are identical and are located with their magnetic axes 120 degrees apart.The winding is arranged in two layers, each full-pitch coil of N_c turns having one side in the top of a slot and the other coil side in the bottom of a slot a pole pitch away.
- Fig. 4.11(b) shows that the mmf wave is a series of steps each of height $2\text{N}_c i_a$. It can be seen that the distributed winding produces a closer approximation to a sinusoidal mmf wave than the concentrated coil of Fig.4.10 does.



(a)

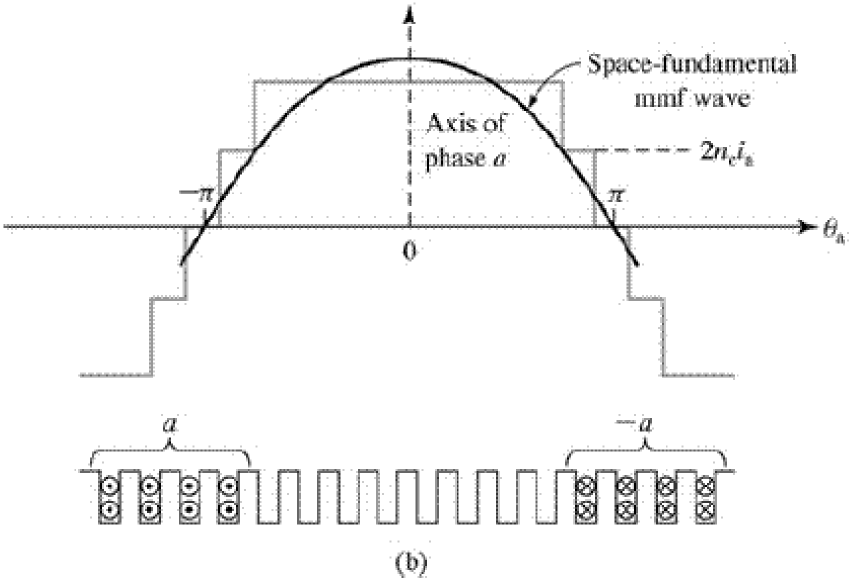


Figure 4.11 The mmf of one phase of a distributed two-pole,

three-phase winding with full-pitch coils.

- The modified form of (4.3) for a distributed multipole winding is

$F_{\text{agl}} = \frac{4}{\pi}(\frac{k_w N_{\text{ph}}}{\text{poles}}) i_a \cos(\frac{\text{poles}}{2}\theta_a)$ (4.5)

N_{ph}: number of series turns per phase,

k_w: winding factor, a reduction factor taking into account the distribution of the winding, typically in the range of 0.85 to 0.95, $k_w = k_b k_p (\text{or} k_d k_p)$.

- The peak amplitude of this mmf wave is

$(F_{\text{agl}})_{\text{peak}} = \frac{4}{\pi}(\frac{k_w N_{\text{ph}}}{\text{poles}}) i_a$ (4.6)

- Eq. (4.5) describes the space-fundamental component of the mmf wave produced by current in phase a of a distributed winding.

- If $i_a = I_m \cos\omega t$ the result will be an mmf wave which is stationary in space and varies sinusoidally both with respect to θ_a and in time.
- The application of three-phase currents will produce a rotating mmf wave.

- Rotor windings are often distributed in slots to reduce the effects of space harmonics.

- Fig. 4.12(a) shows the rotor of a typical two-pole round-rotor generator.
- As shown in Fig. 4.12(b), there are fewer turns in the slots nearest the pole face.
- The fundamental air-gap mmf wave of a multipole rotor winding is

$F_{\text{agl}} = \frac{4}{\pi}(\frac{k_r N_r}{\text{poles}}) I_r \cos(\frac{\text{poles}}{2}\theta_r)$ (4.7)

$(F_{\text{agl}})_{\text{peak}} = \frac{4}{\pi}(\frac{k_r N_r}{\text{poles}}) I_r$ (4.8)

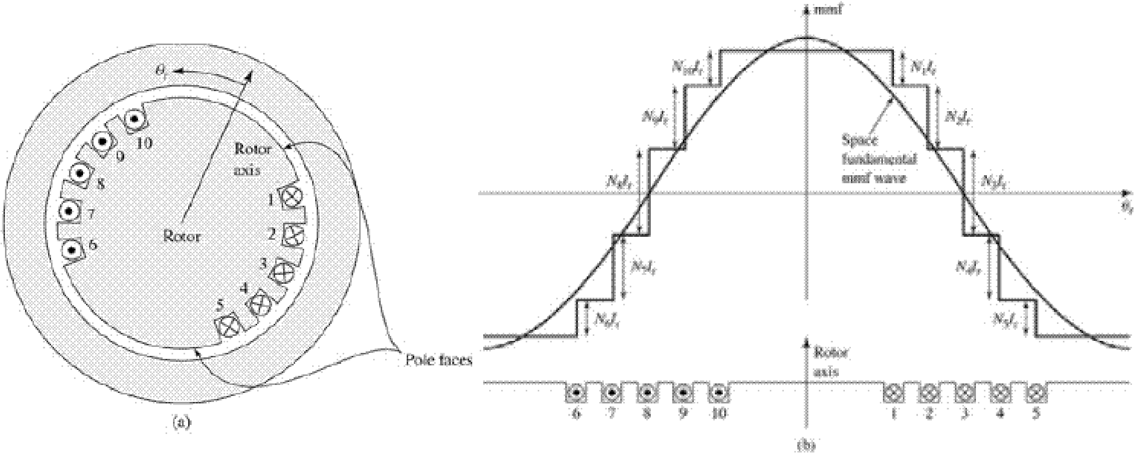


Figure 4.12 The air-gap mmf of a distributed winding on the rotor of a round-rotor generator.

§4.3.2 DC Machines

- Because of the restrictions imposed on the winding arrangement by the commutator, the mmf wave of a dc machine armature approximates a sawtooth waveform more nearly than the sine wave of ac machines.

- Fig. 4.13 shows diagrammatically in cross section the armature of a two-pole dc machine.

- The armature coil connections are such that the armature winding produces a magnetic field whose axis is vertical and thus is perpendicular to the axis of the field winding.
- As the armature rotates, the magnetic field of the armature remains vertical due to commutator action and a continuous unidirectional torque results.

- The mmf wave is illustrated and analyzed in Fig. 4.14.

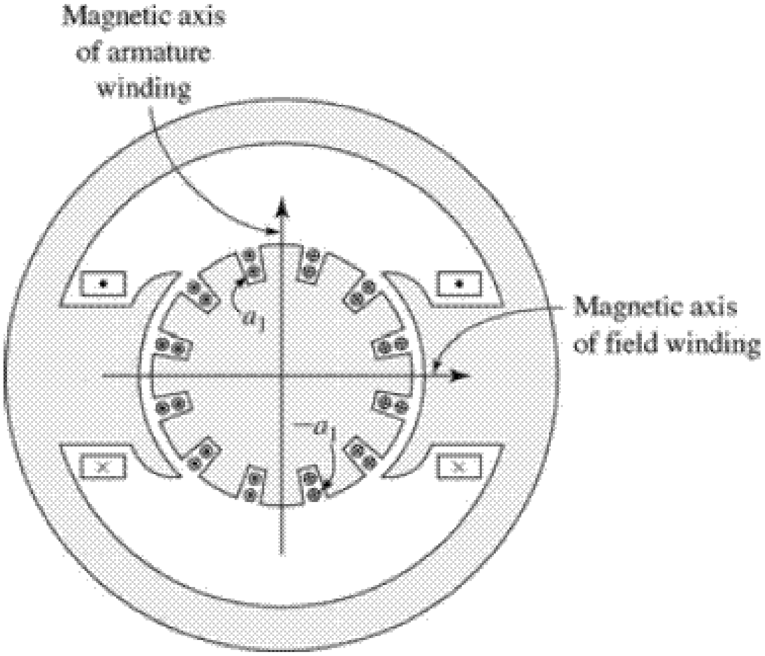
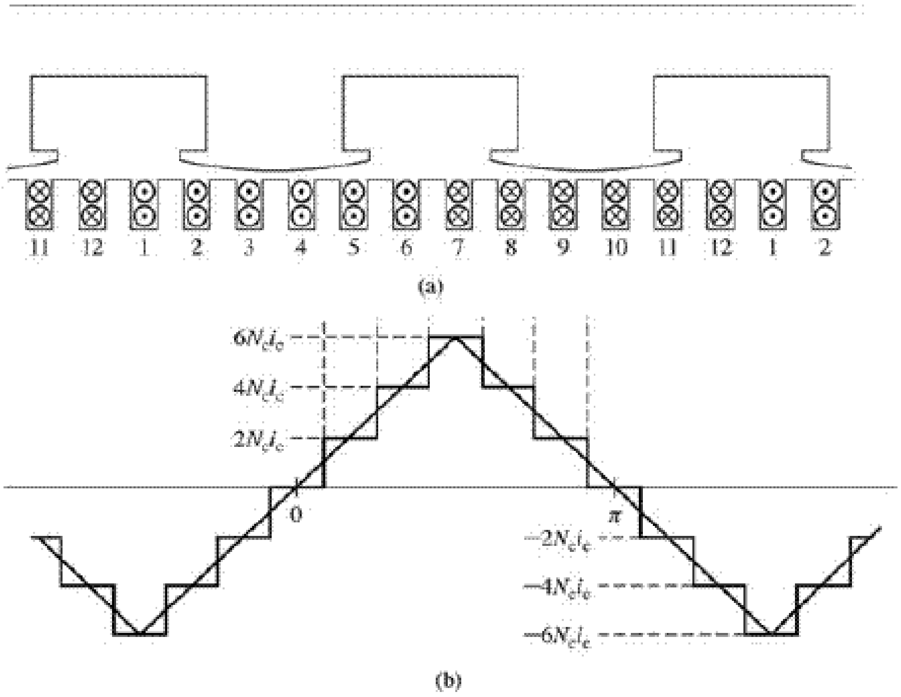

Magnetic axis of armature winding
a_1
$-a_1$
Magnetic axis of field winding

Figure 4.13 Cross section of a two-pole dc machine.



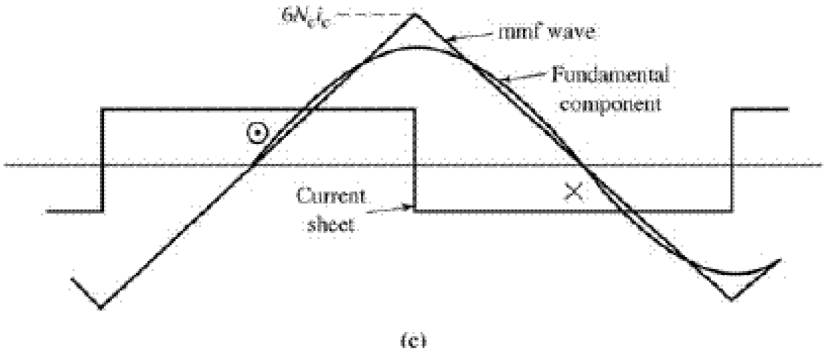


Figure 4.14 (a) Developed sketch of the dc machine of Fig. 4.22; (b) mmf wave; (c) equivalent sawtooth mmf wave, its fundamental component, and equivalent rectangular current sheet.

DC machines often have a magnetic structure with more than two poles.

- Fig. 4.15(a) shows schematically a four-pole dc machine.
- The machine is shown in laid-out form in Fig. 4.15(b).

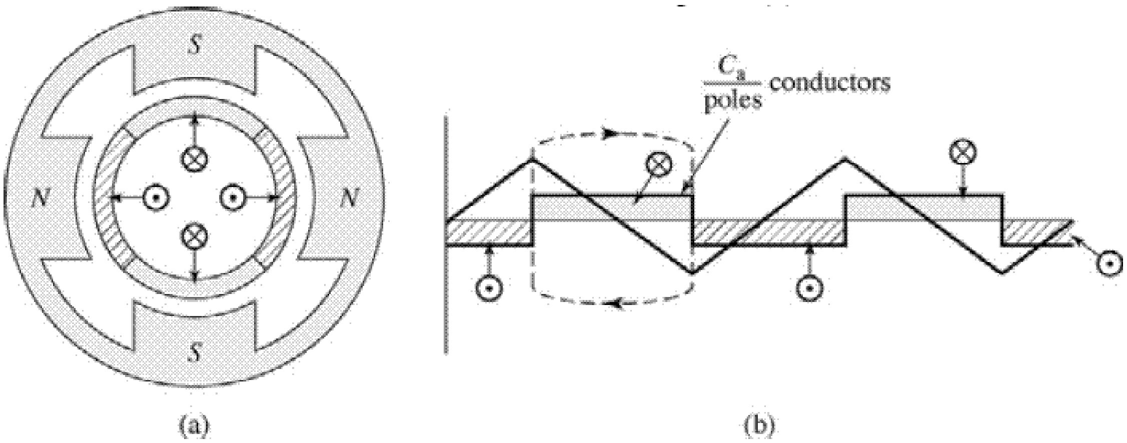


Figure 4.15 (a) Cross section of a four-pole dc machine; (b) development of current sheet and mmf wave.

- The peak value of the sawtooth armature mmf wave can be written as

$$(F_{\text{ag}})_{\text{peak}} = \left(\frac{C_a}{2\text{m.poles}}\right) i_a \text{A.turn/ pole} \quad (4.9)$$

Ca = total number of conductors in armature winding

m = number of parallel paths through armature winding ia = armature current, A

$(F_{\mathrm{ag}})_{\mathrm{peak}} = (\frac{N_a}{\mathrm{poles}})i_a$,$N_a = C_a/(2\mathrm{m})$:no.of series armature turns (4.10)

$(F_{\mathrm{ag}})_{\mathrm{peak}} = \frac{8}{\pi^2}(\frac{N_a}{\mathrm{poles}})i_a$ (4.11)

§4.4 Magnetic Fields In Rotating Machinery

- The behavior of electric machinery is determined by the magnetic fields created by currents in the various windings of the machine.

- The investigations of both ac and dc machines are based on the assumption of sinusoidal spatial distribution of mmf.
- Results from examining a two-pole machine can immediately be extrapolated to a multipole machine.

§4.4.1 Magnetic with Uniform Air Gaps

- Consider machines with uniform air gaps.

- Fig. 4.16(a) shows a single full-pitch, N-turn coil in a high-permeability magnetic structure $\mu \to \infty$, with a concentric, cylindrical rotor.

- In Fig. 4.16(b) the air-gap mmf $F_{\mathrm{ag}}$is plotted versus angle θ_a.
- Fig. 4.16(c) demonstrates the air-gap constant radial magnetic field H_{ag}.

$H_{\mathrm{ag}} = \frac{F_{\mathrm{ag}}}{g}$ (4.12)

$(H_{\mathrm{agl}}) = \frac{F_{\mathrm{agl}}}{g} = \frac{4}{\pi}(\frac{\mathrm{Ni}}{2\mathrm{g}})\cos\theta_a$ (4.13)

$(H_{\mathrm{agl}})_{\mathrm{peak}} = \frac{4}{\pi}(\frac{\mathrm{Ni}}{2\mathrm{g}})$ (4.14)

- For a distributed winding, the air-gap magnetic field intensity is

$H_{\mathrm{agl}} = \frac{4}{\pi}(\frac{k_w N_{\mathrm{ph}}}{g.\mathrm{poles}})i_a\cos(\frac{\mathrm{poles}}{2}\theta_a)$ (4.15)

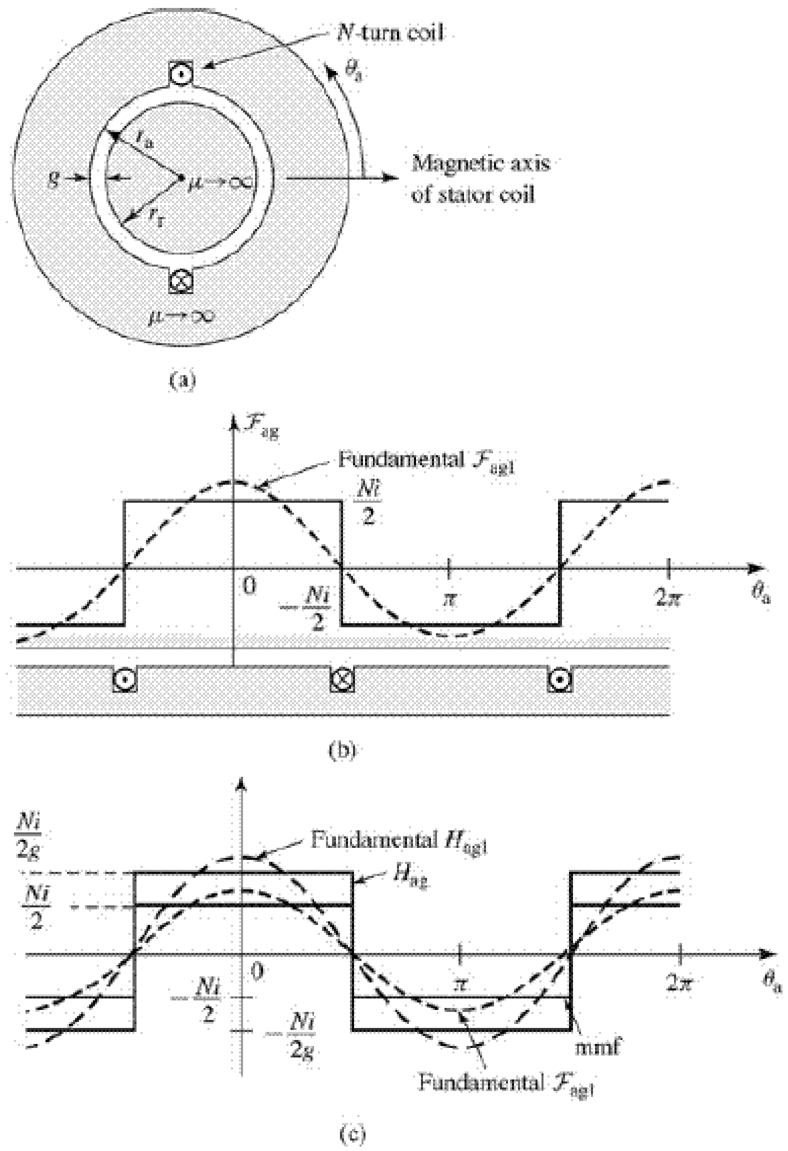


Figure 4.16 The air-gap mmf and radial component of H_{ag} for a concentrated full-pitch winding.

§4.4.2 Machines with Nonuniform Air Gaps

- The air-gap magnetic-field distribution of machines with nonuniform air gaps is more complex than that of uniform-air-gap machines.
- Fig. 4.17(a) shows the structure of a typical dc machine and Fig. 4.17 (b) shows the structure of a typical salient-pole synchronous machine.

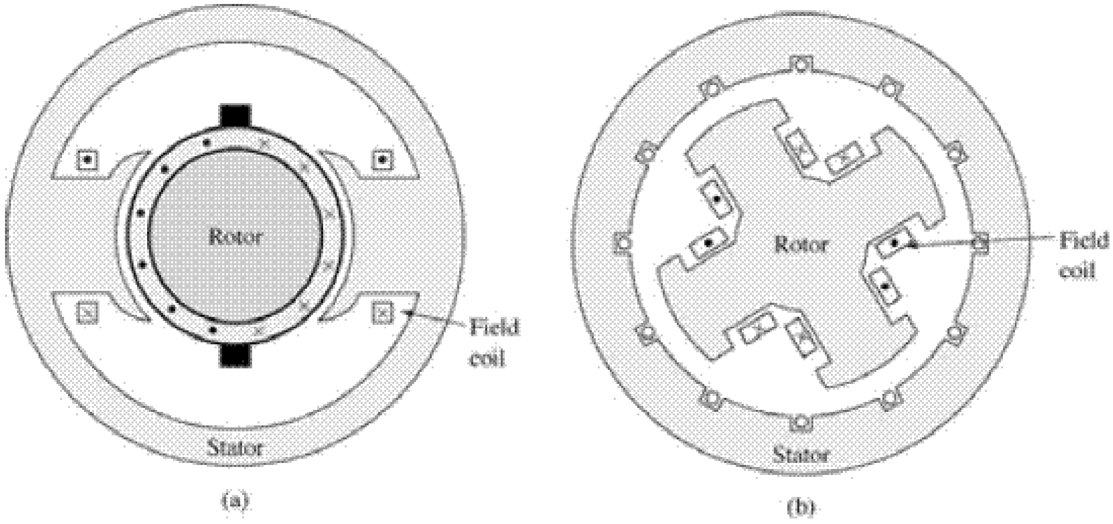


Figure 4.17 Structure of typical salient-pole machines:

(a) dc machine and (b) salient-pole synchronous machine.

- Detailed analysis of the magnetic field distributions requires complete solutions of the field problem.
- Fig. 4.18 shows the magnetic field distribution in a salient-pole dc generator (obtained by finite-element solution).

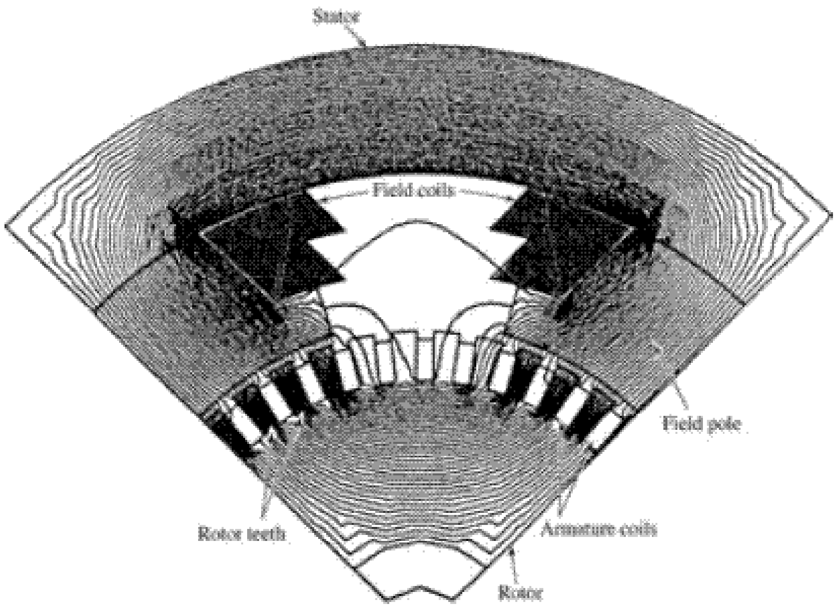


Figure 4.18 Finite-element solution of the magnetic field distribution in a salient-pole dc generator. Field coils excited; no current in armature coils.

(General Electric Company.)

§4.5 Rotating MMF Waves in AC Machines

- To understand the theory and operation of polyphase ac machines, it is necessary to study the nature of the mmf wave produced by a polyphase winding.

§4.5.1 MMF Wave of a Single-Phase Winding

- Fig. 4.19(a) shows the space-fundamental mmf distribution of a single-phase winding.
- Note that from Eq. (4.5), F_{agl} is

$$F_{\text{agl}} = \frac{4}{\pi}(\frac{k_w N_{\text{ph}}}{\text{poles}})i_a \cos(\frac{\text{poles}}{2}\theta_a) \quad (4.16)$$

When the winding is exicted by a current

$$i_a = I_a \cos\omega_e t \quad (4.17)$$

the mmf distribution is given by

$$\begin{aligned} F_{\text{agl}} &= F_{\text{max}}\cos(\frac{\text{poles}}{2}\theta_a)\cos\omega_e t \\ &= F_{\text{max}}\cos(\theta_{\text{ae}})\cos\omega_e t \end{aligned} \quad (4.18)$$

$$F_{\text{max}} = \frac{4}{\pi}(\frac{k_w N_{\text{ph}}}{\text{poles}})I_a \quad (4.19)$$

-
  - This mmf distribution remains fixed in space with an amplitude that varies sinusoidally in time at frequency ω_c , as shown in Fig. 4.19(a).
- The air-gap mmf of a single-phase winding exicted by a source of ac current can be resolved into rotating traveling waves.
  - By the identity $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \cos(\alpha + \beta)$

$$F_{\text{agl}} = F_{\text{max}}[\frac{1}{2}\cos(\theta_{\text{ae}} - \omega_e t) + \frac{1}{2}\cos(\theta_{\text{ae}} + \omega_e t)] \quad (4.20)$$

$$F_{\text{agl}}^{+} = F_{\text{max}}\cos(\theta_{\text{ae}} - \omega_e t) \quad (4.21)$$

$F_{\mathrm{agl}}^{-} = \frac{1}{2} F_{\max} \cos(\theta_{\mathrm{ae}} + \omega_e t)$ (4.22)

- F_{agl}travels in the $+\theta_a$direction and F_{agl}travels in the $-\theta_a$ direction.
- This decomposition is shown graphically in Fig. 4.19(b) and in a phasor representation in Fig. 4.19(c).

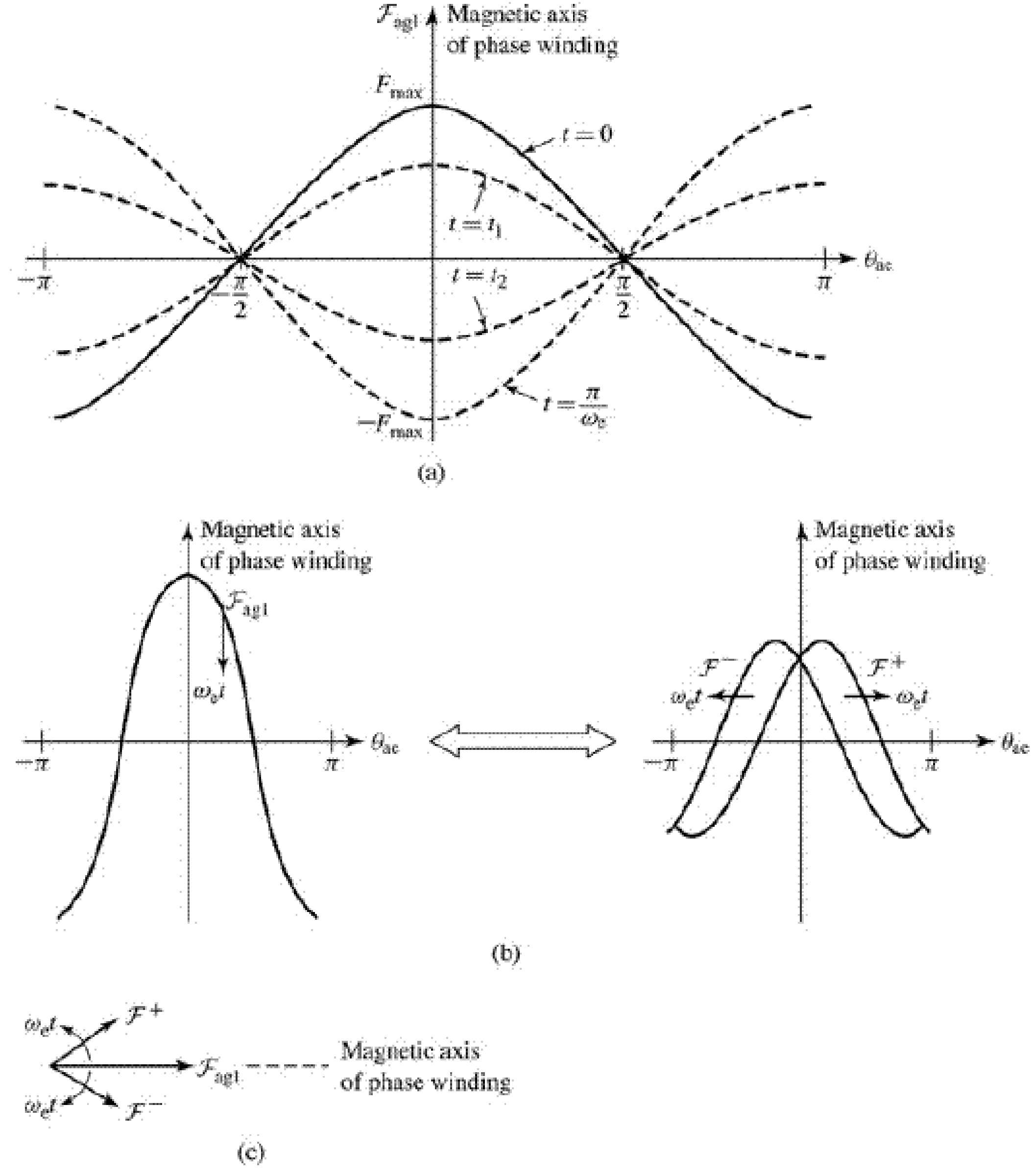


Figure 4.19 Single-phase-winding space-fundamental air-gap mmf: (a) mmf distribution of a

single-phase winding at various times; (b) total mmf F_{ag1}decomposed into two traveling waves F and F^{+} ; (c) phasor decomposition of F.

§4.5.2 MMF Wave of a Polyphase Winding

- We are to study the mmf distribution of three-phase windings such as those found on the stator of three-phase induction and synchronous machines.

In a three-phase machine, the windings of the individual phases are displaced from each other by 120 electrical degrees in space around the air-gap circumference as shown in Fig.4.20 in which the concentrated full-pitch coils may be considered to represent distributed windings.

- Under balanced three-phase conditions, the excitation currents (Fig. 4.20) are

$i_a = I_m \cos \omega_e t$ (4.23)

$i_b = I_m \cos(\omega_e t - 120^o)$ (4.24)

$i_c = I_m \cos(\omega_e t + 120^o)$ (4.25)

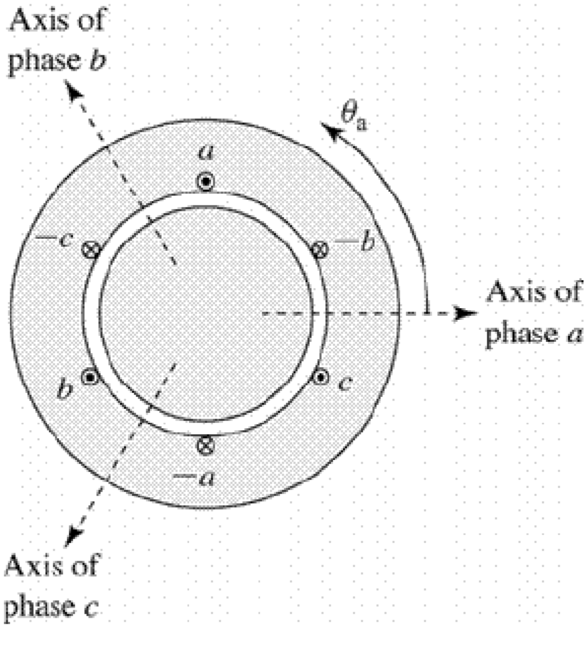


Figure 4.20 Simplified two-pole three-phase stator winding.

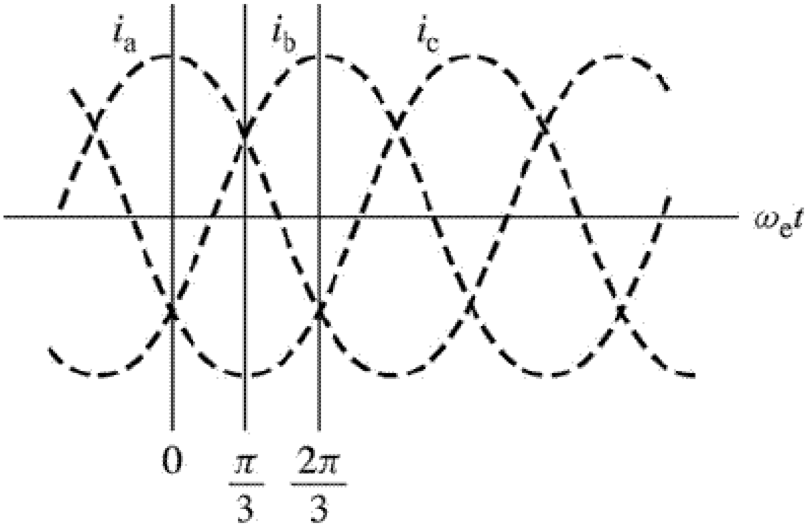


Figure 4.21 Instantaneous phase currents under balanced three-phase conditions.

- The mmf of phase a has been shown to be

$F_{a1} = F_{a1}^{+} + F_{a1}^{-}$ (4.26)

$F_{a1}^{+} = \frac{1}{2} F_{\text{max}} \cos(\theta_{\text{ae}} - \omega_e t)$ (4.27)

$F_{a1}^{-} = \frac{1}{2} F_{\text{max}} \cos(\theta_{\text{ae}} + \omega_e t)$ (4.28)

$F_{\text{max}} = \frac{4}{\pi} (\frac{k_w N_{\text{ph}}}{\text{poles}}) I_m$ (4.29)

- Similarly, for phases b and c

$F_{b1} = F_{b1}^{+} + F_{b1}^{-}$ (4.30)

$F_{b1}^{+} = \frac{1}{2} F_{\text{max}} \cos(\theta_{\text{ae}} - \omega_e t)$ (4.31)

$F_{b1}^{-} = \frac{1}{2} F_{\text{max}} \cos(\theta_{\text{ae}} + \omega_e t + 120^o)$(4.32)

$F_{c1} = F_{c1}^{+} + F_{c1}^{-}$ (4.33)

$F_{c1}^{+} = \frac{1}{2} F_{\text{max}} \cos(\theta_{\text{ae}} - \omega_e t)$ (4.34)

$F_{c1}^{-} = F_{\text{max}} \cos(\theta_{\text{ae}} + \omega_e t - 120^o)$(4.35)

- The total mmf is the sum

$F(\theta_{\mathrm{ae}},t) = F_{a1} + F_{b1} + F_{c1}$(4.36)

It can be performed in terms of the positive- and negative- traveling waves.

$$\begin{gathered} F^{-}(\theta_{\mathrm{ae}},t) = F_{a1}^{-} + F_{b1}^{-} + F_{c1}^{-} \\ = \frac{1}{2}F_{\mathrm{max}}[\cos(\theta_{\mathrm{ae}} + \omega_e t) + \cos(\theta_{\mathrm{ae}} + \omega_e t - 120^o) + \cos(\theta_{\mathrm{ae}} + \omega_e t + 120^o)] \\ = 0 \end{gathered}$$

$$\begin{aligned} F^{+}(\theta_{\mathrm{ae}},t) &= F_{a1}^{+} + F_{b1}^{+} + F_{c1}^{+} \\ &= \frac{3}{2}F_{\mathrm{max}}\cos(\theta_{\mathrm{ae}} - \omega_e t) \end{aligned} \quad (4.37)$$

- The result of displacing the three windings by 120^o in space phase and displacing the winding currents by 120^o in time phase is a single positive-traveling mmf wave

$$\begin{aligned} F(\theta_{\mathrm{ae}},t) &= \frac{3}{2}F_{\mathrm{max}}\cos(\theta_{\mathrm{ae}} - \omega_e t) \\ &\frac{3}{2}F_{\mathrm{max}}\cos((\frac{\mathrm{poles}}{2})\theta_a - \omega_e t) \end{aligned} \quad (4.38)$$

- Under balanced three-phase conditions, the three-phase winding produces an air-gap mmf wave which rotates at synchronous angular velocity ω_s (rad/sec)

$\omega_s = (\frac{2}{\mathrm{poles}})\omega_e$ (4.39)

ω_c: angular velocity of the applied electrical excitation (rad/sec)

- [missing_resource: graphics31.wmf]: synchronous speed

$f_e = \omega_e/(2\pi)$: applied electrical frequency

$n_s = (\frac{120}{\mathrm{poles}})f_e r/\mathrm{min}$ (4.40)

- A polyphase winding exicted by balanced polyphase currents produces a rotating mmf wave.

- It is the interaction of this magnetic flux wave with that of the rotor which produces torque.
- Constant torque is produced when rotor-produced magnetic flux rotates in synchronism with that of the stator.

§4.5.3 Graphical Analysis of Polyphase MMF

- For balanced three-phase currents, the production of a rotating mmf can also be shown graphically.

- Refer to Fig. 4.22.

- As time passes, the resultant mmf wave retains its sinusoidal form and amplitude but rotates progressively around the air gap.
- The net result is an mmf wave of constant amplitude rotating at uniform angular velocity.

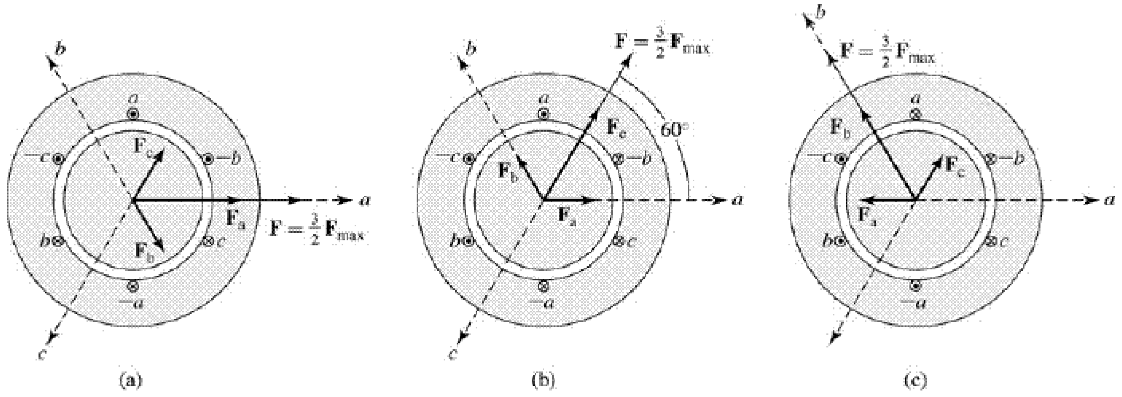


Figure 4.22 The production of a rotating magnetic field by means of three-phase currents.

§4.6 Generated Voltage

§4.6.1 AC Machines

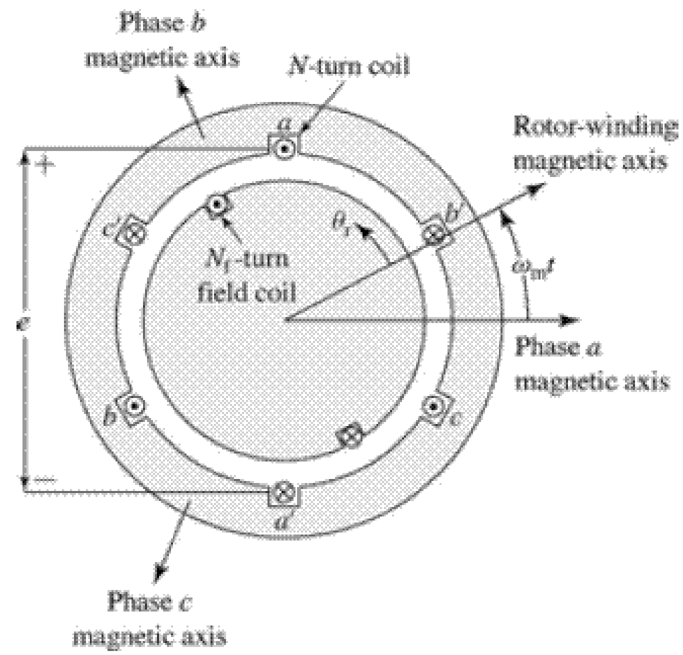


Figure 4.23 Cross-sectional view of an elementary three-phase ac machine.

$$B_{\text{peak}} = \frac{4\mu_o}{\pi g}\left(\frac{k_f N_f}{\text{poles}}\right) I_f \quad (4.41)$$

$$B = B_{\text{peak}} \cos\left(\frac{\text{poles}}{2}\theta_r\right) \quad (4.42)$$

$$\begin{aligned} \Phi &= l \int_{-\pi/\text{poles}}^{+\pi/\text{poles}} B_{\text{peak}} \cos\left(\frac{\text{poles}}{2}\theta_r\right) r d\theta_r \\ &= \left(\frac{2}{\text{poles}}\right) 2 B_{\text{peak}} l r \end{aligned} \quad (4.43)$$

$$\begin{aligned} \lambda_a &= k_w N_{\text{ph}} \Phi_p \cos\left(\left(\frac{\text{poles}}{2}\right)\omega_m t\right) \\ &= k_w N_{\text{ph}} \Phi_p \cos \omega_{\text{me}} t \end{aligned} \quad (4.44)$$

$$\omega_{\text{me}} = \left(\frac{\text{poles}}{2}\right)\omega_m \quad (4.45)$$

$$e_a = \frac{d\lambda_a}{dt} = k_w N_{\text{ph}} \frac{d\Phi_p}{dt} \cos \omega_{\text{me}} t - \omega_{\text{me}} k_w N_{\text{ph}} \Phi_p \sin \omega_{\text{me}} t \quad (4.46)$$

$$e_a = -\omega_{\text{me}} k_w N_{\text{ph}} \Phi_p \sin \omega_{\text{me}} t \quad (4.47)$$

$$E_{\max} = \omega_{\text{me}} k_w N_{\text{ph}} \Phi_p = 2\pi f_{\text{me}} k_w N_{\text{ph}} \Phi_p \quad (4.48)$$

$$E_{\text{rms}} = \frac{2\pi}{\sqrt{2}} f_{\text{me}} k_w N_{\text{ph}} \Phi_p = \sqrt{2} f_{\text{me}} k_w N_{\text{ph}} \Phi_p \quad (4.49)$$

$$E_a = \frac{1}{\pi}\int_0^\pi \omega_{\mathrm{me}} N\Phi_p \mathrm{sin}(\omega_{\mathrm{me}}t)d(\omega_{\mathrm{me}}t) = \frac{2}{\pi}\omega_{\mathrm{me}} N\Phi_p \text{ (4.50)}$$

$$E_a = (\frac{\mathrm{poles}}{\pi}) N\Phi_p\omega_m = \mathrm{polesN}\Phi_p(\frac{n}{30}) \text{ (4.51)}$$

$$E_a = (\frac{\mathrm{poles}}{2\pi})(\frac{C_a}{m})\Phi_p\omega_m = (\frac{\mathrm{poles}}{60})(\frac{C_a}{m})\Phi_p n \text{ (4.52)}$$

§4.7 Torque in Nonsalient-pole Machines

- Consider the elementary smooth-air-gap machine of Fig.4.24 with one winding on the stator and one on the rotor and with θ_m being the mechanical angle between the axes of the two windings. These windings are distributed over a number of slots so that their mmf waves can be approximated by space sinusoids. In Fig.4.24a the coil sides s, -s and r, -r mark the positions of the centers of the belts of conductors comprising the distributed windings. An alternative way of drawing these windings is shown in Fig.4.24b, which also shows reference directions for voltages and currents. Here it is assumed that current in the arrow direction produces a magnetic field in the air gap in the arrow direction, so that a single arrow defines reference directions for both current and flux.

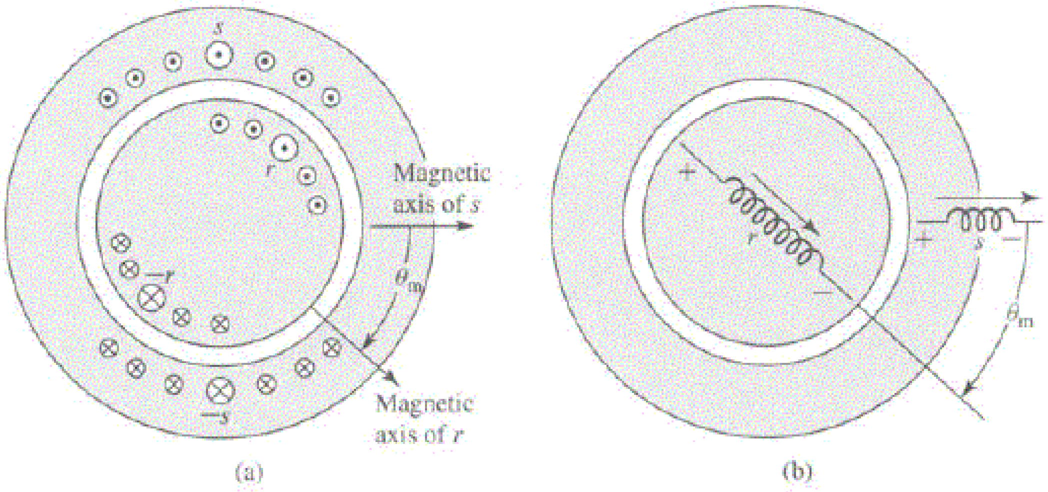


Figure 4.24 Elementary two-pole machine with smooth air gap: (a) winding distribution and (b) schematic representation.

- The stator and rotor are concentric cylinders, and slot openings are neglected. Consequently, our elementary model does not include the effects of salient poles, which are investigated in later chapters. We also assume that the reluctances of the stator and rotor iron are negligible. Finally, although Fig.4.34 shows a two-pole machine, we will write the derivations that follow for the general case of a multipole machine, replacing θ_m by the electrical rotor angle.

$$\theta_{\text{me}} = \left[\frac{\text{poles}}{2}\right]\theta_m \text{ (4.53)}$$

- Based upon these assumptions, the stator and rotor self-inductances L_{ss} and L_{rr} can be seen to be constant, but the stator-to-rotor mutual inductance depends on the electrical angle θ_{me} between the magnetic axes of the stator and rotor windings. The mutual inductance is at its positive maximum when θ_{me}=0 or 2 π, is zero when $\theta_{\text{me}} = \pm\pi/2$, and is at its negative maximum when $\theta_{\text{me}} = \pm\pi$. On the assumption of sinusoidal mmf waves and a uniform air gap, the space distribution of the air-gap flux wave is sinusoidal, and the mutual inductance will be of the form

$$L_{\text{sr}}(\theta_{\text{me}}) = L_{\text{sr}}\text{cos}(\theta_{\text{me}}) \text{ (4.54)}$$

where the script letter E denotes an inductance which is a function of the electrical angle θ_{me}. The italic capital letter L denotes a constant value. Thus Lsr is the magnitude of the mutual inductance; its value when the magnetic axes of the stator and rotor are aligned (θ_{me}= 0). In terms of the inductances, the stator and rotor flux linkages) $\lambda_s$and $\lambda_r$are

$$\lambda_s = L_{\text{ss}} + L_{\text{sr}}(\theta_{\text{me}})i_r = L_{\text{ss}}i_s + L_{\text{sr}}\text{cos}(\theta_{\text{me}})i_r \text{ (4.55)}$$

$$\lambda_r = L_{\text{sr}}(\theta_{\text{me}})i_s + L_{\text{rr}}i_r = L_{\text{sr}}\text{cos}(\theta_{\text{me}})i_s + L_{\text{rr}}i_r \text{ (4.56)}$$

In matrix notation

$$\begin{bmatrix}\lambda_s\\ \lambda_r\end{bmatrix} = \begin{bmatrix}L_{\text{ss}} & L_{\text{sr}}(\theta_{\text{me}})\\ L_{\text{sr}}(\theta_{\text{me}}) & L_{\text{rr}}\end{bmatrix}\begin{bmatrix}i_s\\ i_r\end{bmatrix} \text{ (4.57)}$$

The terminal voltages $v_s$and $v_r$are

$$v_s = R_s i_s + \frac{d\lambda_s}{\text{dt}} \text{ (4.58)}$$

$$v_r = R_r i_r + \frac{d\lambda_r}{\text{dt}} \text{ (4.59)}$$

where $R_s$and R_r are the resistances of the stator and rotor windings respectively.

- When the rotor is revolving, θ_{me} must be treated as a variable. Differentiation of Eqs.4.56 and 4.57 and substitution of the results in Eqs.4.59 and 4.60 then give

$$v_s = R_s i_s + L_{\mathrm{ss}} \frac{\mathrm{di}_s}{\mathrm{dt}} + L_{\mathrm{sr}} \cos(\theta_{\mathrm{me}}) \frac{\mathrm{di}_r}{\mathrm{dt}} - L_{\mathrm{sr}} i_r \sin(\theta_{\mathrm{me}}) \frac{d\theta_{\mathrm{me}}}{\mathrm{dt}} \text{ (4.60)}$$

$$v_r = R_r i_r + L_{\mathrm{rr}} \frac{\mathrm{di}_r}{\mathrm{dt}} + L_{\mathrm{sr}} \cos(\theta_{\mathrm{me}}) \frac{\mathrm{di}_r}{\mathrm{dt}} - L_{\mathrm{sr}} i_s \sin(\theta_{\mathrm{me}}) \frac{d\theta_{\mathrm{me}}}{\mathrm{dt}} \text{ (4.61)}$$

$$\frac{d\theta_{\mathrm{me}}}{\mathrm{dt}} = \omega_{\mathrm{me}} = \left[\frac{\mathrm{poles}}{2}\right] \omega_m \text{ (4.62)}$$

is the instantaneous speed in electrical radians per second. In a two-pole machine, θme and ωme are equal to the instantaneous shaft angle θm and the shaft speed ωm respectively. In a multipole machine, they are related by Eqs.4.54 and 4.46. The second and third terms on the fight-hand sides of Eqs.4.61 and 4.62 are L(di/dt) induced voltages like those induced in stationary coupled circuits such as the windings of transformers. The fourth terms are caused by mechanical motion and are proportional to the instantaneous speed. These are the speed voltage terms which correspond to the interchange of power between the electric and mechanical systems.

- The electromechanical torque can be found from the coenergy.

$$\begin{aligned} W^{'}_{\mathrm{fld}} &= \frac{1}{2} L_{\mathrm{ss}} i_s^2 + \frac{1}{2} L_{\mathrm{rr}} i_r^2 + L_{\mathrm{sr}} i_s i_r \cos\theta_{\mathrm{me}} \\ &= \frac{1}{2} L_{\mathrm{ss}} i_s^2 + \frac{1}{2} L_{\mathrm{rr}} i_r^2 + L_{\mathrm{sr}} i_s i_r \cos\left[\left(\frac{\mathrm{poles}}{2}\right)\theta_m\right] \end{aligned} \text{ (4.63)}$$

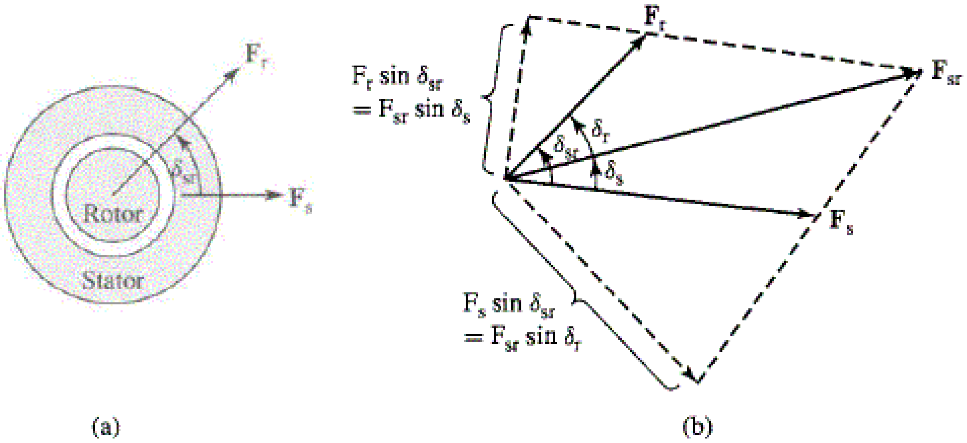
Note that the coenergy of Eq.4.63 has been expressed specifically in terms of the shaft angle θ_m because the torque expression requires that the torque be obtained from the derivative of the coenergy with respect to the spatial angle θ_m and not with respect to the electrical angle θ_{me}. Thus,

$$\begin{aligned} T = \frac{\partial W^{'}_{\mathrm{fld}}(i_s, i_r, \theta_m)}{\partial \theta_m} \Big|_{i_s, i_r} &= -\left[\frac{\mathrm{poles}}{2}\right] L_{\mathrm{sr}} i_s i_r \sin\left[\frac{\mathrm{poles}}{2} \theta_m\right] \\ &= -\left[\frac{\mathrm{poles}}{2}\right] L_{\mathrm{sr}} i_s i_r \sin\theta_{\mathrm{me}} \end{aligned} \text{ (4.64)}$$

where T is the electromechanical torque acting to accelerate the rotor (i.e., a positive torque acts to increase θ_m). The negative sign in Eq.4.64 means that the electromechanical torque acts in the direction to bring the magnetic fields of the stator and rotor into alignment.

4.7.2 Magnetic Field Viewpoint

- In the discussion of Section 4.7.1 the characteristics of a rotating machine as viewed from its electric and mechanical terminals have been expressed in terms of its winding inductances. This viewpoint gives little insight into the physical phenomena which occur within the machine. In this section, we will explore an alternative formulation in terms of the interacting magnetic fields.



(a) (b)

Figure 4.25 Simplified two-pole machine: (a) elementary model and

(b) vector diagram of mmf waves. Torque is produced by the tendency of the rotor and stator magnetic fields to align. Note that these figures are drawn with δ_{sr} positive, i.e., with the rotor mmf wave
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leading that of the stator F_s.

- As we have seen, currents in the machine windings create magnetic flux in the air gap between the stator and rotor, the flux paths being completed through the stator and rotor iron. This condition corresponds to the appearance of magnetic poles on both the stator and the rotor, centered on their respective magnetic axes, as shown in Fig.4.35a for a two-pole machine with a smooth air gap. Torque is produced by the tendency of the two component magnetic fields to line up their magnetic axes.

- We shall derive an expression for the magnetic coenergy stored in the air gap in terms of the stator and rotor mmfs and the angle δ_{sr} between their magnetic axes. The torque can then be found from the partial derivative of the coenergy with respect to angle δ_{sr}.
- The line integral of Hag across the gap then is simply $H_{\mathrm{ag}}g$ and equals the resultant air-gap mmf F_{sr} produced by the stator and rotor windings; thus

$H_{\mathrm{ag}}g = F_{\mathrm{sr}}$ (4.65)

- The mmf waves of the stator and rotor are spatial sine waves with δ_{sr} being the phase angle between their magnetic axes in electrical degrees. They can be represented by the space vectors F_s and F_r drawn along the magnetic axes of the stator- and rotor mmf waves respectively. The resultant mmf F_{sr} acting across the air gap, also a sine wave, is their vector sum.

$F_{\mathrm{sr}}^2 = F_s^2 + F_r^2 + 2\mathrm{F}_s F_r \cos\delta_{\mathrm{sr}}$ (4.66)

The resultant radial H_{ag}field is a sinusoidal space wave whose peak value $H_{\mathrm{ag,peak}}$ is, from Eq.4.65,

$(H_{\mathrm{ag}})_{\mathrm{peak}} = \frac{F_{\mathrm{sr}}}{g}$ (4.67)

Average coenergy density = $\frac{\mu_o}{2}\left[\frac{(H_{\mathrm{ag}})_{\mathrm{peak}}^2}{2}\right] = \frac{\mu_o}{4}\left[\frac{F_{\mathrm{sr}}}{g}\right]^2$ (4.68)

$$\begin{aligned} W_{\mathrm{fld}}^{'} &= (\text{average coenergy density})(\text{volume of air gap}) \\ &= \frac{\mu_o}{4}\left[\frac{F_{\mathrm{sr}}}{g}\right]^2 \pi D\mathrm{lg} = \frac{\mu_o \pi \mathrm{Dl}}{4\mathrm{g}} F_{\mathrm{sr}}^2 \end{aligned} \qquad (4.69)$$

$W_{\mathrm{fld}}^{'} = \frac{\mu_o \pi \mathrm{Dl}}{4\mathrm{g}}\left(F_s^2 + F_r^2 + 2\mathrm{F}_s F_r \cos\delta_{\mathrm{sr}}\right)$ (4.70)

For a two-pole machine

$T = \frac{\partial W_{\mathrm{fld}}^{'}}{\partial \delta_{\mathrm{sr}}}\ |_{F_s,F_r} = -\left[\frac{\mu_o \pi \mathrm{Dl}}{2\mathrm{g}}\right] F_s F_r \sin\delta_{\mathrm{sr}}$ (4.71)

The general expression for the torque for a multipole machine is

$$T=-\left[\frac{\text{poles}}{2}\right]\left[\frac{\mu_o\pi\text{Dl}}{2\text{g}}\right]F_sF_r\sin\delta_{\text{sr}}\ (4.72)$$

- In this equation, δ_{sr} is the electrical space-phase angle between the rotor and stator mmf waves and the torque T acts in the direction to accelerate the rotor. Thus when δ_{sr} is positive, the torque is negative and the machine is operating as a generator.
- Similarly, a negative value of δ_{sr} corresponds to positive torque and, correspondingly, motor action. The torque, acting to accelerate the rotor, can then be expressed in terms of the resultant mmf wave F_{sr}; thus

$$T=-\left[\frac{\text{poles}}{2}\right]\left[\frac{\mu_o\pi\text{Dl}}{2\text{g}}\right]F_sF_{\text{sr}}\sin\delta_s\ (4.73)$$

$$T=-\left[\frac{\text{poles}}{2}\right]\left[\frac{\mu_o\pi\text{Dl}}{2\text{g}}\right]F_rF_{\text{sr}}\sin\delta_r\ (4.74)$$

$$T=\left[\frac{\text{poles}}{2}\right]\left[\frac{\pi\text{Dl}}{2}\right]B_{\text{sr}}F_r\sin\delta_r\ (4.75)$$

- One of the inherent limitations in the design of electromagnetic apparatus is the saturation flux density of magnetic materials. Because of saturation in the armature teeth the peak value B_{sr} of the resultant flux-density wave in the air gap is limited to about 1.5 to 2.0T. The maximum permissible value of the winding current, and hence the corresponding mmf wave, is limited by the temperature rise of the winding and other design requirements. Because the resultant flux density and mmf appear explicitly in Eq.4.75, this equation is in a convenient form for design purposes and can be used to estimate the maximum torque which can be obtained from a machineof a given size.